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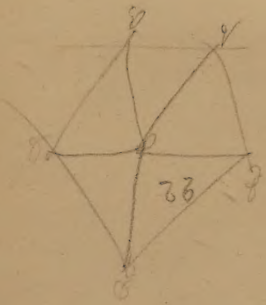

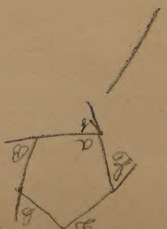

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EL. GEOM.

PREFACE.

Those who are conversant with the preparation of elementary text-books, have experienced the difficulty of adapting them to the various wants which they are intended to supply.

The institutions of education are of all grades, from the college to the district school, and although there is a wide difference between the extremes, the level, in passing from one grade to the other, is scarcely broken.

Each of these classes of seminaries requires text-books adapted to its own peculiar wants; and if each held its proper place in its own class, the task of supplying suitable works would not be difficult.

An indifferent college is generally inferior, in the system and scope of its instruction, to the academy or high school; while the district school is often found to be superior to its neighboring academy.

The Geometry of Legendre, embracing a complete course of Geometrical science, is all that is desired in the colleges and higher seminaries; while the Practical Mathematics for Practical Men, recently published, is designed to meet the wants of those schools which are strictly elementary and practical in their systems of instruction.

But still a large class of seminaries remained unsupplied with a suitable text-book on Elementary Geometry and Trigonometry: viz., those where the pupils are carried beyond the acquisition of facts and mere practical knowledge, but have not time to go through with a full course of mathematical studies.

It is for such, that the following work is designed. It has been the aim of the author to present the striking and important truths of Geometry in a form more simple and concise than could be adopted in a complete treatise, and yet to preserve the exactness of rigorous reasoning.

In this system of Geometry nothing has been taken for granted, and nothing passed over without being fully demonstrated.

The Trigonometry, including the applications to the measurements of heights and distances, has been written upon the same plan and for the same objects: it embraces all the important theorems and all the striking examples.

In order, however, to render the applications of Geometry to the mensuration of surfaces and solids complete in itself, a few rules have been given which are not demonstrated. This forms an exception to the general plan of the work, but being added in the form of an appendix, it does not materially break its unity.

That the work may be useful in advancing the interests of education, is the hope and ardent wish of the author.

Fishkill Landing,
May, 1851

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ELEMENTARY

GEOMETRY.

BOOK I.

DEFINITIONS AND REMARKS.

1. *Extension* has three dimensions, length, breadth, and thickness.

Geometry is the science which has for its object:

1st. The measurement of extension; and 2dly, To discover, by means of such measurement, the properties and relations of geometrical figures.

2. A *Point* is that which has place, or position, but not magnitude.

3. A *Line* is length, without breadth or thickness.

4. A *Straight Line* is one which lies in the same direction between any two of its points.

5. A *Curve Line* is one which changes is direction at every point.

The word *line* when used alone, will designate a straight line; and the word *curve*, a curve line.

6. A *Surface* is that which has length and breadth, without height or thickness.

7. A *Plane Surface* is that which lies even throughout its whole extent, and with which a straight line, laid in any direction, will exactly coincide in its whole length.

8. A *Curved Surface* has length and breadth without thickness, and like a curve line is constantly changing its direction

9. A *Solid* or *Body* is that which has length, breadth, and thickness. Length, breadth, and thickness are called dimen-

sions. Hence, a solid has three dimensions, a surface two, and a line one. A point has no dimensions, but position only

10. *Geometry* treats of lines, surfaces, and solids.

11. A *Demonstration* is a course of reasoning which establishes a truth.

12. An *Hypothesis* is a supposition on which a demonstration may be founded.

13. A *Theorem* is something to be proved by demonstration.

14. A *Problem* is something proposed to be done.

15. A *Proposition* is something proposed either to be done or demonstrated—and may be either a problem or a theorem.

16. A *Corollary* is an obvious consequence, deduced from something that has gone before.

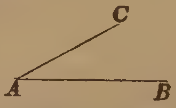
17. A *Scholium* is a remark on one or more preceding propositions.

18. An *Axiom* is a self evident proposition.

OF ANGLES.

19. An *Angle* is the portion of a plane included between two straight lines which meet at a common point. The two straight lines are called the *sides* of the angle, and the common point of intersection, the *vertex*.

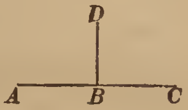
Thus, the part of the plane included between *AB* and *AC* is called an *angle:* *AB* and *AC* are its *sides*, and *A* its *vertex*.



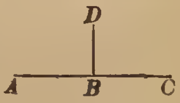
An angle is generally read, by placing the letter at the vertex in the middle. Thus, we say, the angle *CAB*. We may, however, say simply, the angle *A*.

20. One line is said to be perpendicular to another when it inclines no more to the one side than to the other

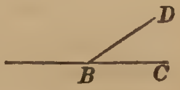
The two angles formed are then equal to each other. Thus, if the line *DB* is perpendicular to *AC*, the angle *DBA* will be equal to *DBC*.



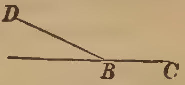
21. When two lines are perpendicular to each other, the angles which they form are called right angles. Thus, *DBA* and *DBC* are called right angles.



22. An acute angle is less than a right angle. Thus, *DBC* is an acute angle.

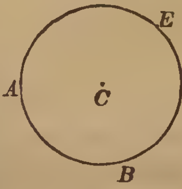


23. An obtuse angle is greater than a right angle. Thus, *DBC* is an obtuse angle.



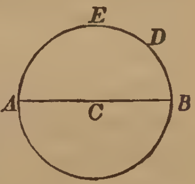
24. The circumference of a circle is a curve line all the points of which are equally distant from a certain point within called the centre.

Thus, if all the points of the curve *AEB* are equally distant from the centre *C*, this curve will be the circumference of a circle.

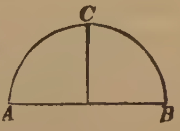


25. Any portion of the circumference, as *AED*, is called an *arc*.

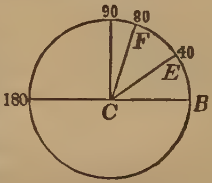
26. The diameter of a circle is a straight line passing through the centre and terminating at the circumference. Thus, *ACB* is a diameter.



27. One half of the circumference, as *ACB* is called a *semicircumference;* and one quarter of the circumference, as *AC*, is called a *quadrant*

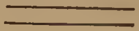


28. The circumference of a circle is used for the measurement of angles. For this purpose it is divided into 360 equal parts called degrees, each degree into 60 equal parts called minutes, and each minute into 60 equal parts called seconds. The degrees, minutes, and seconds are marked thus ° ′ ″; and 9° 18′ 16″, are read, 9 degrees 18 minutes and 16 seconds.

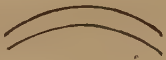


29. Let us suppose the circumference of a circle to be divided into 360 degrees, beginning at the point *B*. If through the point of division marked 40, we draw *CE*, then, the angle *ECB* will be equal to 40 degrees. If *CF* were drawn through the point of division marked 80, the angle *BCF* would be equal to 80 degrees.

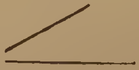
OF LINES.



30. Two straight lines are said to be *parallel*, when being produced either way, as far as we please, they will not meet each other.



31. Two curves are said to be parallel or *concentric*, when they are the same distance from each other at every point.



32. Oblique lines are those which approach each other, and meet if sufficiently produced.

33. Lines which are parallel to the horizon, or to the water level, are called horizontal lines.

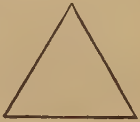
34. Lines which are perpendicular to the horizon, or to the water level, are called vertical lines.

OF PLANE FIGURES.

35. A Plane Figure is a portion of a plane terminated on all sides by lines, either straight or curved.

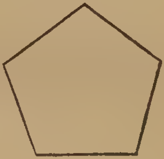
36. If the lines which bound a figure are straight, the space which they inclose is called a *rectilineal* figure, or *polygon* The lines themselves, taken together, are called the *perimeter* of the polygon. Hence, the perimeter of a polygon is the sum of all its sides.

37. A polygon of three sides is called a triangle.

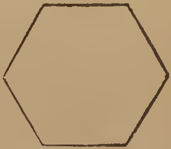


38. A polygon of four sides is called a quadrilateral.

39. A polygon of five sides is called a pentagon.



40. A polygon of six sides is called a hexagon.



41. A polygon of seven sides is called a heptagon.

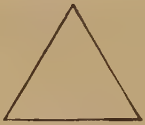
42. A polygon of eight sides is called an octagon.

43. A polygon of nine sides is called a nonagon.

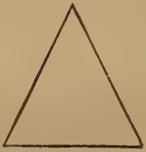
44. A polygon of ten sides is called a decagon.

45. A polygon of twelve sides is called a dodecagon.

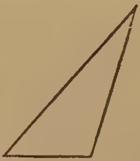
46. There are several kinds of triangles.



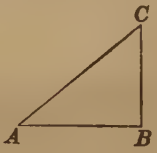
First. An equilateral triangle, which has its three sides all equal.



Second. An isosceles triangle, which has two of its sides equal.

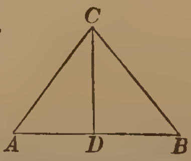


Third. A scalene triangle, which has its three sides all unequal.



Fourth. A right angled triangle, which has one right angle.

In the right angled triangle *ABC*, the side *AC*, opposite the right angle, is called the hypothenuse.

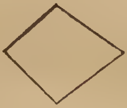


47. The base of a triangle is the side on which it stands. Thus, *AB* is the base of the triangle *ACB*.

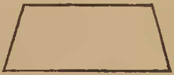
The altitude of a triangle is a line drawn from the angle opposite the base and perpendicular to the base. Thus, *CD* is the altitude of the triangle *ACB*.

48. There are three kinds of quadrilaterals.

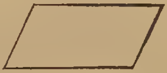
1. The *trapezium*, which has none of its sides parallel.



2. The *trapezoid*, which has only two of its sides parallel.

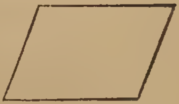


3. The *parallelogram*, which has its opposite sides parallel.

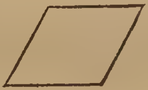


49. There are four kinds of parallelograms:

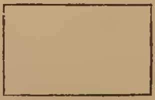
1. The *rhomboid*, which has no right angle.



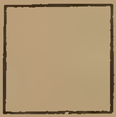
2. The *rhombus*, or *lozenge*, which is an equilateral rhomboid.

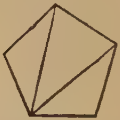


3. The *rectangle*, which is an equiangular parallelogram.



4. The *square*, which is both equilateral and equiangular.





50. A DIAGONAL of a figure is a line which joins the vertices of two angles not adjacent.

51. The base of a figure is the side on which it is supposed to stand; and the altitude is a line drawn from the opposite side or angle, perpendicular to the base.

AXIOMS.

1. Things which are equal to the same thing are equal to each other.

2. If equals be added to equals, the wholes will be equal.

3. If equals be taken from equals, the remainders will be equal.

4. If equals be added to unequals, the wholes will be unequal.

5. If equals be taken from unequals, the remainders will be unequal.

6. Things which are double of equal things, are equal to each other.

7. Things which are halves of the same thing, are equal to each other.

8. The whole is greater than any of its parts

9. The whole is equal to the sum of all its parts.

10. All right angles are equal to each other.

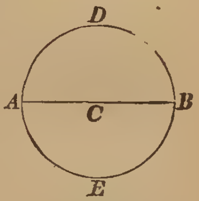
11. A straight line is the shortest distance between two points.

12. Magnitudes, which being applied to each other, coincide throughout their whole extent, are equal.

PROPERTIES OF POLYGONS.

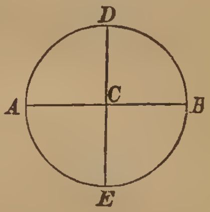
THEOREM I.

Every diameter of a circle divides the circumference into two equal parts.



Let *ADBE* be the circumference of a circle, and *ACB* a diameter: then will the part *ADB* be equal to the part *AEB*.

For, suppose the part *AEB* to be turned around *AB*, until it shall fall on the part *ADB*. The curve *AEB* will then exactly coincide with the curve *ADB*, or else there would be some point in the curve *AEB* or *ADB*, unequally distant from the centre *C*, which is contrary to the definition of a circumference (Def. 24). Hence, the two curves will be equal (Ax. 12).

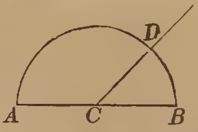


Corollary 1. If two lines, *AB*, *DE*, be drawn through the centre *C* perpendicular to each other, each will divide the circumference into two equal parts; and the entire circumference will be divided into the equal quadrants *DB*, *DA*, *AE*, and *EB*.

Cor. 2. Hence, a right angle, as *DCB*, is measured by one quadrant, or 90 degrees; two right angles by a semicircumference, or 180 degrees; and four right angles by the whole circumference, or 360 degrees

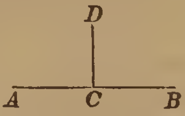
THEOREM II.

If one straight line meet another straight line, the sum of the two adjacent angles will be equal to two right angles.

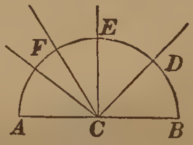


Let the straight line *CD* meet the straight line *AB*, at the point *C*; then will the angle *DCB* plus the angle *DCA* be equal to two right angles.

About the centre *C*, with any radius as *CB*, suppose a semicircumference to be described. Then, the angle *DCB* will be measured by the arc *BD*, and the angle *DCA* by the arc *AD*. But the sum of the two arcs is equal to a semicircumference: hence, the sum of the two angles is equal to two right angles (Th. i, Cor. 2).

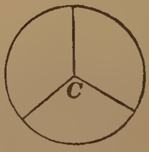


Cor. 1. If one of the angles, as *DCB*, is a right angle, the other angle, *DCA* will also be a right angle.



Cor. 2. Hence, all the angles which can be formed at any point *C*, by any number of lines, *CD*, *CE*, *CF*, &c., drawn on the same side of *AB*, are equal to two right angles: for, they will be measured by a semicircumference.

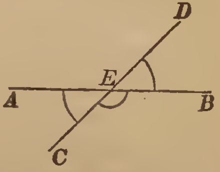
Cor. 3. If *DC* meets two lines *CB*, *CA*, making *DCB* plus *DCA* equal to two right angles, *ACB* will form one straight line.



Cor. 4. Hence, also, all the angles which can be formed round any point, as *C*, are equal to four right angles. For, the sum of all the arcs which measure them, is equal to the entire circumference, which is the measure of four right angles (Th. i. Cor. 2).

THEOREM III.

If two straight lines intersect each other, the opposite or vertical angles which they form, are equal.



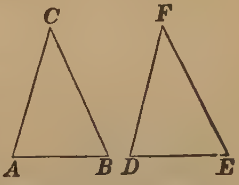
Let the two straight lines AB and CD intersect each other at the point E: then will the opposite angle AEC be equal to DEB, and $AED = CEB$.

For, since the line AE meets the line CD, the angle $AEC + AED =$ two right angles. But since the line DE meets the line AB, we have $DEB + AED =$ two right angles. Taking away from these equals the common angle AED, and there will remain the angle AEC equal to the angle DEB (Ax. 3).

In the same manner we may prove that the angle AED is equal to the angle CEB.

THEOREM IV.

If two triangles have two sides and the included angle of the one, equal to two sides and the included angle of the other, each to each, the two triangles will be equal.

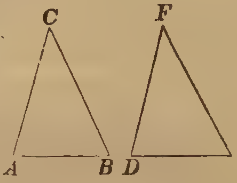


Let the triangles ABC and DEF have the side AC equal to DF, CB to FE, and the angle C equal to the angle F: then will the triangle ACB be equal to the triangle DEF.

For, suppose the side AC, of the triangle ACB, to be placed on DF, so that the extremity C shall fall on the extremity F: then, since the sides are equal, A will fall on D.

But since the angle C is equal to the angle F, the line CB

will fall on *FE*; and since *CB* is equal to *FE*, the extremity *B* will fall on *E*; and consequently the side *AB* will fall on the side *DE* (Ax. 11). Hence, the two triangles will fill the same space, and consequently are equal (Ax. 12.).

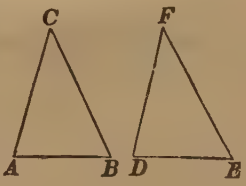


Scholium. Two triangles are said to be equal, when being applied the one to the other they exactly coincide (Ax. 12). Hence, *equal* triangles have their like parts equal, each to each, since those parts coincide with each other. The converse of the proposition is also true, namely, that *two triangles which have all the parts of the one equal to the corresponding parts of the other, each to each, are equal:* for if applied the one to the other, the equal parts will coincide.

THEOREM V.

If two triangles have two angles and the included side of the one, equal to two angles and the included side of the other, each to each, the two triangles will be equal.

Let the two triangles *ABC* and *DEF* have the angle *A* equal to the angle *D*, the angle *B* equal to the angle *E*, and the included side *AB* equal to the included side *DE*: then will the triangle *ABC* be equal to the triangle *DEF*.



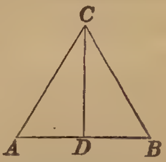
For, let the side *AB* be placed on the side *DE*, the extremity *A* on the extremity *D*; and since the sides are equal, the point *B* will fall on the point *E*.

Then since the angle *A* is equal to the angle *D*, the side

AC will take the direction *DF:* and since the angle *B* is equal to the angle *E*, the side *BC* will fall or the side *EF:* hence, the point *C* will be found at the same time on *DF* and *EF*, and therefore will fall at the intersection *F:* consequently, all the parts of the triangle *ABC* will coincide with the parts of the triangle *DEF*, and therefore, the two triangles are equal

THEOREM VI.

In an isosceles triangle the angles opposite the equal sides are equal to each other.



Let *ABC* be an isosceles triangle, having the side *AC* equal to the side *CB:* then will the angle *A* be equal to the angle *B*.

For, suppose the line *CD* to be drawn dividing the angle *C* into two equal parts.

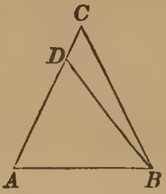
Then, the two triangles *ACD* and *DCB*, have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each: that is, the side *AC* equal to *BC*, the side *CD* common, and the included angle *ACD* equal to the included angle *DCB:* hence the two triangles are equal (Th. iv); and hence, the angle *A* is equal to the angle *B*.

Cor. 1. Hence, the line which bisects the vertical angle of an isosceles triangle, bisects the base. It is also perpendicular to the base, since the angle *CDA* is equal to the angle *CDB*.

Cor. 2. Hence, also, every equilateral triangle, must also be equiangular: that is, have all its angles equal, each to each

THEOREM VII.

Conversely.—*If a triangle has two of its angles equal, the sides opposite those angles will also be equal.*



In the triangle ABC, let the angle A be equal to the angle B: then will the side BC be equal to the side AC.

For, if the two sides are not equal, one of them must be greater than the other. Suppose AC to be the greater side. Then take a part AD equal to BC

Now, in the two triangles ADB and ABC, we have the side $AD=BC$, by hypothesis; the side AB common, and the angle A equal to the angle B: hence, the two triangles have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each: hence, the two triangles are equal (Th. iv), that is, a part ADB is equal to the whole ABC, which is impossible (Ax. 8): consequently, the side AC cannot be greater than the side CB, and hence, the triangle is isosceles.

Scholium 1. The method of reasoning pursued in the last theorem, is called the "reductio ad absurdum," or a proof that leads to a known absurdity.

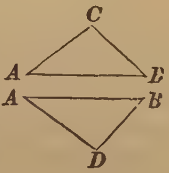
Let us analyze this method of reasoning. We wished to prove that the two sides AC, CB were equal. We supposed them unequal, and AC the greater—that was an hypothesis (See Def. 12). We then reasoned on the hypothesis, and proved a part equal to the whole, which we know to be false (Ax. 8) Hence, we conclude that the hypothesis is untrue, because after a correct chain of reasoning it leads to a result which we know to be absurd

Scholium 2. Generally,—If the demonstration is based on known principles, previously proved, or admitted in the axioms, the conclusion will always be true. But, if the demonstration is based on an hypothesis, (as in the last theorem, that AC was the greater side), and the conclusion is *contrary* to what has been previously proved, or admitted in the axioms then, it follows, that the hypothesis cannot be true.

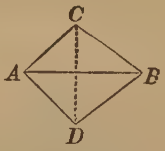
The former is called a *direct*, and the latter an *indirect* demonstration.

THEOREM VIII.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, the three angles will also be equal, each to each.



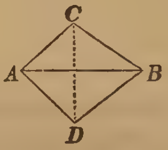
Let the two triangles ABC, ABD, have the side AB equal to the side AB, the side AC equal to AD, and the side CB equal to DB: then will the corresponding angles also be equal, viz: the angle A will be equal to the angle A, the angle B to the angle B, and the angle C to the angle D.



For, suppose the triangles to be joined by their longest equal sides AB, and the line CD to be drawn.

Then, since the side AC is equal to AD, by hypothesis, the triangle ADC will be isosceles; and therefore, the angle ACD will be equal to the angle ADC (Th. vi). In like manner, in the triangle CBD, the side CB is equal to DB: hence, the angle BCD is equal to the angle BDC.

Now, by the addition of equals, we have



$$ACD+BCD=ADC+BDC$$

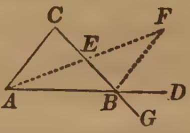
that is, the angle $ACB=ADB$.

Now, the two triangles ACB and ADB have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each: hence, the remaining angles will be equal (Th. iv): consequently, the angle CAB is equal to BAD, and the angle CBA to the angle ABD.

Sch. The angles of the two triangles which are equal to each other, are those which lie opposite the equal sides.

THEOREM IX.

If one side of a triangle is produced, the outward angle is greater than either of the inward opposite angles.



Let ABC be a triangle, having the side AB produced to D: then will the outward angle CBD be greater than either of the inward opposite angles A or C.

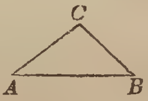
For, suppose the side CB to be bisected at the point E. Draw AE, and produce it until EF is equal to AE, and then draw BF.

Now, since the two triangles AEC and BEF have $AE=EF$ and $EC=EB$, and the included angle AEC equal to the included angle BEF (Th. iii), the two triangles will be equal in all respects (Th. iv): hence, the angle EBF will be equal to the angle C. But the angle CBD is greater than the angle CBF, consequently it is greater than the angle C.

In like manner, if CB be produced to G, and AB be bisected, it may be proved that the outward angle ABG, or its equal CBD (Th. iii), is greater than the angle A.

THEOREM X.

The sum of any two sides of a triangle is greater than the third side.

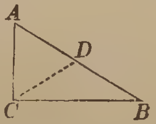


Let ABC be a triangle: then will the sum of two of its sides, as AC, CB, be greater than the third side AB.

For the straight line AB is the shortest distance between the two points A and B (Ax. xi): hence, $AC+CB$ is greater than AB.

THEOREM XI.

The greater side of every triangle is opposite the greater angle: and conversely, the greater angle is opposite the greater side.



First. In the triangle CAB, let the angle C be greater than the angle B: then, will the side AB be greater than the side AC.

For, draw CD, making the angle BCD equal to the angle B. Then, the triangle CBD will be isosceles: hence, the side $CD=DB$ (Th. vii.)

But, by the last theorem AC is less than $AD+CD$; that is. less than $AD+DB$, and consequently less than AB.

Secondly. Let us suppose the side AB to be greater than AC; then will the angle C be greater than the angle B.

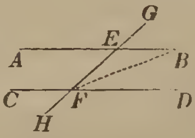
For if the angle C were equal to B, the triangle CAB would be isosceles, and the side AC would be equal to AB (Th vii) which would be contrary to the hypothesis.

Again if the angle C were less than B, then, by the first part of the theorem, the side AB would be less than AC, which is also contrary to the hypothesis Hence, since C

cannot be equal to B, nor less than B, it follows that it must be greater

THEOREM XII.

If a straight line intersect two parallel lines, the alternate angles will be equal.



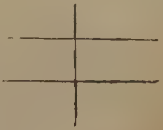
If two parallel straight lines, AB CD, are intersected by a third line GH, the angles AEF and EFD are called *alternate* angles. It is required to prove that these angles are equal.

If they are unequal one of them must be greater than the other. Suppose EFD to be the greater angle.

Now conceive FB to be drawn, making the angle EFB equal to the angle AEF, and meeting AE in B.

Then, in the triangle FEB the outward angle FEA is greater than either of the inward angles B or EFB (Th. ix.); and therefore, EFB can never be equal to AEF so long as FB meets EB.

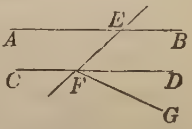
But since we have supposed EFD to be greater than AEF, it follows that EFB could not be equal to AEF, if FB fell below FD. Therefore, if the angle EFB is equal to the angle AEF, FB cannot meet AB, nor fall below FD, and consequently must coincide with the parallel CD (Def. 30): and hence, the alternate angles AEF and EFD are equal.



Cor. If a line be perpendicular to one of two parallel lines, it will also be perpendicular to the other

THEOREM XIII.

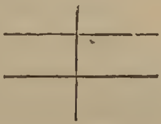
Conversely,—*If a line intersect two straight lines, making the alternate angles equal, those straight lines will be parallel.*



Let the line *EF* meet the lines *AB*, *CD*, making the angle *AEF* equal to the angle *EFD*: then will the lines *AB* and *CD* be parallel.

For, if they are not parallel, suppose through the point *F* the line *FG* to be drawn parallel to *AB*.

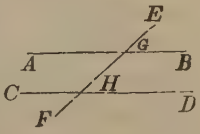
Then, because of the parallels *AB*, *FG*, the alternate angles, *AEF* and *EFG* will be equal (Th. xii). But, by hypothesis, the angle *AEF* is equal to *EFD*: hence, the angle *EFD* is equal to the angle *EFG* (Ax. 1); that is, a part is equal to the whole, which is absurd (Ax. 8): therefore, no line but *CD* can be parallel to *AB*.



Cor. If two lines are perpendicular to the same line, they will be parallel to each other.

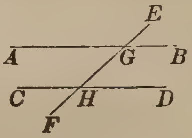
THEOREM XIV.

If a line cut two parallel lines, the outward angle is equal to the inward opposite angle on the same side; and the two inward angles, on the same side, are equal to two right angles.



Let the line *EF* cut the two parallels *AB* *CD*: then will the outward angle *EGB* be equal to the inward opposite angle *EHD*; and the two inward angles, *BGH* and *GHD*, will be equal to two right angles.

First. Since the lines *AB*, *CD*, are parallel, the angle *AGH* is equal to the alternate angle *GHD* (Th. xii); but the angle *AGH* is equal to the opposite angle *EGB*: hence, the angle *EGB* is equal to the angle *EHD* (Ax. 1).



Secondly. Since the two adjacent angles *EGB* and *BGH* are equal to two right angles (Th. ii); and since the angle *EGB* has been proved equal to *EHD*, it follows that the sum of *BGH* plus *GHD*, is also equal to two right angles.

Cor. 1. Conversely, if one straight line meets two other straight lines, making the angles on the same side equal to each other, those lines will be parallel.

Cor. 2. If a line intersect two other lines, making the sum of the two inward angles equal to two right angles, those two lines will be parallel.

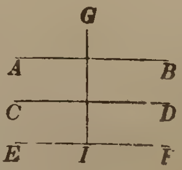
Cor. 3. If a line intersect two other lines, making the sum of the two inward angles less than two right angles, those lines will not be parallel, but will meet if sufficiently produced.

THEOREM XV.

All straight lines which are parallel to the same line, are parallel to each other.

Let the lines *AB* and *CD* be each parallel to *EF*: then will they be parallel to each other.

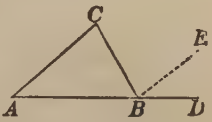
For, let the line *GI* be drawn perpendicular to *EF*: then will it also be perpendicular to the parallels *AB*, *CD* (Th. xii Cor.).



Then, since the lines AB and CD are perpendicular to the line GI, they will be parallel to each other (Th. xiii. Cor).

THEOREM XVI.

If one side of a triangle be produced, the outward angle will be equal to the sum of the inward opposite angles.



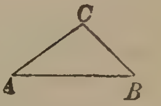
In the triangle ABC, let the side AB be produced to D: then will the outward angle CBD be equal to the sum of the inward opposite angles A and C.

For, conceive the line BE to be drawn parallel to the side AC. Then, since BC meets the two parallels AC, BE, the alternate angles ACB and CBE will be equal (Th. xii).

And since the line AD cuts the two parallels BE and AC the angles EBD and CAB are equal to each other (Th. xiv) Therefore, the inward angles C and A, of the triangle ABC are equal to the angles CBE and EBD; and consequently the sum of the two angles, A and C, is equal to the outward angle CBD (Ax. 1).

THEOREM XVII.

In any triangle the sum of the three angles is equal to two right angles.



Let ABC be any triangle: then will the sum of the three angles

$A+B+C=$two right angles.

For, let the side AB be produced to D Then, the outward angle

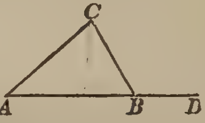
$CBD=A+C$ (Th. xvi).

To each of these equals add the angle CBA, and we shall have

$$CBD+CBA=A+C+B.$$

But the sum of the two angles CBD and CBA, is equal to two right angles (Th. ii): hence

$$A+B+C=\text{two right angles (Ax. 1).}$$



Cor. 1. If two angles of one triangle be equal to two angles of another triangle, the third angles will also be equal (Ax. 3).

Cor. 2. If one angle of one triangle be equal to one angle of another triangle, the sum of the two remaining angles in each triangle, will also be equal (Ax. 3).

Cor. 3. If one angle of a triangle be a right angle, the sum of the other two angles will be equal to a right angle; and each angle singly, will be acute.

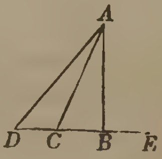
Cor. 4. No triangle can have more than one right angle, nor more than one obtuse angle; otherwise, the sum of the three angles would exceed two right angles: hence, at least two angles of every triangle must be acute.

THEOREM XVIII.

I. *A perpendicular is the shortest line that can be drawn from a given point to a given line.*

II. *If any number of lines be drawn from the same point, those which are nearest the perpendicular are less than those which are more remote.*

Let A be a given point, and DE a straight line. Suppose AB to be drawn perpendicular to DE, and suppose the oblique lines AC and AD also to be



drawn: Then, AB will be shorter than either of the oblique lines, and AC will be less than AD

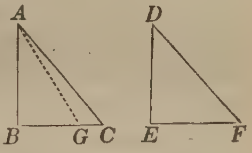
First. Since the angle B, in the triangle ACB, is a right angle, the angle C will be acute (Th. xvii. Cor. 3): and since the greater side of every triangle is opposite the greater angle (Th. xi), the side AC will be greater than AB.

Secondly. Since the angle ACB is acute, the adjacent angle ACD will be obtuse (Th. ii): consequently, the angle D is acute (Th. xvii. Cor. 3), and therefore less than the angle ACD. And since the greater side of every triangle is opposite the greater angle, it follows that AD is greater than AC.

Cor. A perpendicular is the shortest distance from a point to a line.

THEOREM XIX.

If two right angled triangles have the hypothenuse and a side of the one equal to the hypothenuse and a side of the other, the remaining parts will also be equal, each to each.



Let the two right angled triangles ABC and DEF, have the hypothenuse AC equal to DF, and the side AB equal to DE: then will the remaining parts be equal, each to each.

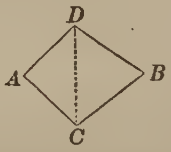
For, if the side BC is equal to EF, the corresponding angles of the two triangles will be equal (Th. viii). If the sides are unequal, suppose BC to be the greater, and take a part, BG, equal to EF, and draw AG.

Then, in the two triangles ABG and DEF, the angle B is equal to the angle E, the side AB to the side DE, and the side BG to the side EF: hence, the two triangles are equal in all respects (Th. iv) and consequently, the side AG is equal to

DF. But DF is equal to AC, by hypothesis; therefore, AG is equal to AC (Ax. 1). But this is impossible (Th. xviii); hence, the sides BC and EF cannot be unequal; consequently, the triangles are equal (Th. viii).

THEOREM XX.

The sum of the four angles of every quadrilateral is equal to four right angles.



Let $ACBD$ be a quadrilateral: then will $A+B+C+D=$four right angles.

Let the diagonal DC be drawn dividing the quadrilateral AB, into two triangles, BDC, ADC.

Then, because the sum of the three angles of each triangle is equal to two right angles (Th. xvii), it follows that the sum of the angles of both triangles is equal to four right angles. But the sum of the angles of both triangles, make up the angles of the quadrilateral. Hence, the sum of the four angles of the quadrilateral is equal to four right angles.

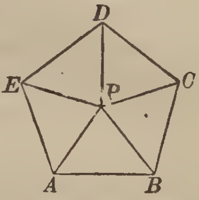
Cor. 1. If then three of the angles be right angles, the fourth angle will also be a right angle.

Cor. 2. If the sum of two of the four angles be equal to two right angles, the sum of the remaining two will also be equal to two right angles.

Cor. 3. Since all the angles of a square or rectangle, are equal to each other (Def. 48), and their sum equal to four right angles, it follows that each angle is equal to one right angle.

THEOREM XXI.

The sum of all the interior angles of any polygon is equal to twice as many right angles, wanting four, as the figure has sides



Let $ABCDE$ be any polygon: then will the sum of its inward angles

$$A+B+C+D+E$$

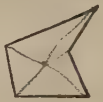
be equal to twice as many right angles, wanting four, as the figure has sides.

For, from any point P, within the polygon, draw the lines PA, PB, PC, PD, PE, to each of the angles, dividing the polygon into as many triangles as the figure has sides.

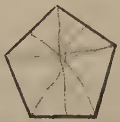
Now, the sum of the three angles of each of these triangles is equal to two right angles (Th. xvii): hence, the sum of the angles of all the triangles is equal to twice as many right angles as the figure has sides.

But the sum of all the angles about the point P is equal to four right angles (Th. ii. Cor. 4); and since this sum makes no part of the inward angles of the polygon, it must be subtracted from the sum of all the angles of the triangles, before found. Hence, *the sum of the interior angles of the polygon is equal to twice as many right angles, wanting four, as the figure has sides.*

Sch. This proposition is not applicable to polygons which have *re-entrant* angles.

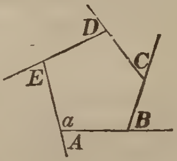


The reasoning is limited to polygons with salient angles, which may properly be named *convex polygons.*



THEOREM XXII.

If every side of a polygon be produced out, the sum of all the outward angles thereby formed, will be equal to four right angles.



Let A, B, C, D, and E, be the outward angles of a polygon formed by producing all the sides. Then will

$A+B+C+D+E=$four right angles.

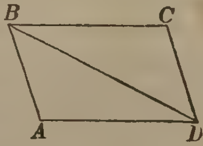
For, each interior angle, plus its exterior angle, as $A+a$, is equal to two right angles (Th. ii). But there are as many exterior as interior angles, and as many of each as there are sides of the polygon: hence, the sum of all the interior and exterior angles will be equal to twice as many right angles as the polygon has sides.

But the sum of all the interior angles together with four right angles, is equal to twice as many right angles as the polygon has sides (Th. xxi): that is, equal to the sum of all the inward and outward angles taken together.

From each of these equal sums take away the inward angles, and there will remain, the outward angles equal to four right angles (Ax. 3).

THEOREM XXIII.

The opposite sides and angles of every parallelogram are equal, each to each: and a diagonal divides the parallelogram into two equal triangles.



Let $ABCD$ be any parallelogram, and DB a diagonal: then will the opposite sides and angles be equal to each other, each to each, and the diagonal DB will divide the parallelogram into two equal triangles.

For, since the figure is a parallelogram, the sides AB, DC are parallel, as also the sides AD, BC. Now, since the parallels are cut by the diagonal DB, the alternate angles will be equal (Th. xii): that is the angle

$$ADB = DBC \quad \text{and} \quad BDC = ABD.$$

Hence the two triangles ADB BDC, having two angles in the one equal to two angles in the other, will have their third angles equal (Th. xvii. Cor. 1), viz. the angle A equal to the angle C, and these are two of the opposite angles of the parallelogram.

Also, if to the equal angles ADB, DBC, we add the equals BDC, ABD, the sums will be equal (Ax. 2): viz. the whole angle ADC to the whole angle ABC, and these are the other two opposite angles of the parallelogram.

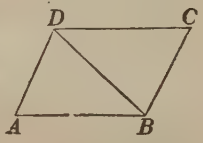
Again, since the two triangles ADB, DBC, have the side DB common, and the two adjacent angles in the one equal to the two adjacent angles in the other, each to each, the two triangles will be equal (Th. v): hence, the diagonal divides the parallelogram into two equal triangles.

Cor. 1. If one angle of a parallelogram be a right angle, each of the angles will also be a right angle, and the parallelogram will be a rectangle.

Cor. 2. Hence, also, the sum of either two adjacent angles of a parallelogram, will be equal to two right angles.

THEOREM XXIV.

If the opposite sides of a quadrilateral, are equal, each to each, the equal sides will be parallel, and the figure will be a parallelogram.



Let $ABCD$ be a quadrilateral, having its opposite sides respectively equal, viz.

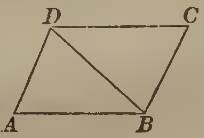
$AB=CD$ and $AD=BC$

then will these sides be parallel, and the figure will be a parallelogram.

For, draw the diagonal BD. Then, the two triangles ABD BDC, have all the sides of the one equal to all the sides of the other, each to each: therefore, the two triangles are equal (Th. viii); hence, the angle ADB, opposite the side AB, is equal to the angle DBC opposite the side DC; therefore, the sides AD, BC, are parallel (Th. xiii). For a like reason DC is parallel to AB, and the figure $ABCD$ is a parallelogram.

THEOREM XXV.

If two opposite sides of a quadrilateral are equal and parallel, the remaining sides will also be equal and parallel, and the figure will be a parallelogram.

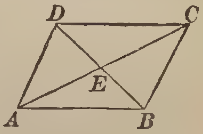


Let $ABCD$ be a quadrilateral, having the sides AB, CD, equal and parallel: then will the figure be a parallelogram.

For, draw the diagonal DB, dividing the quadrilateral into two triangles. Then, since AB is parallel to DC, the alternate angles, ABD and BDC are equal (Th. xii): moreover, the side BD is common; hence the two triangles have two sides and the included angle of the one, equal to two sides and the included angle of the other: the triangles are therefore equal, and consequently AD is equal to BC, and the angle ADB to the angle DBC and consequently, AD is also parallel to BC (Th xiii) Therefore, the figure $ABCD$ is a parallelogram.

THEOREM XXVI.

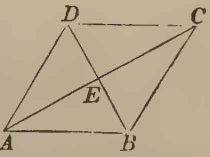
The two diagonals of a parallelogram divide each other into equal parts, or mutually bisect each other.



Let $ABCD$ be a parallelogram, and AC, BD its two diagonals intersecting at E. Then will

$AE=EC$ and $BE=ED$.

Comparing the two triangles AED and BEC, we find the side $AD=BC$ (Th. xxiii), the angle $ADE=EBC$ and $EAD=ECB$: hence, the two triangles are equal (Th. v): therefore, AE, the side opposite ADE, is equal to EC, the side opposite EBC; and ED is equal to EB



Sch. In the case of a rhombus (Def. 48), the sides AB, BC being equal, the triangles AEB and BEC have all the sides of the one equal to the corresponding sides of the other, and are therefore equal. Whence it follows that the angles AEB and BEC are equal. Therefore, the diagonals of a rhombus bisect each other at right angles.

GEOMETRY.

BOOK II,

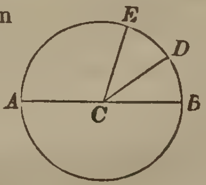
OF THE CIRCLE

DEFINITIONS.

1. The circumference of a circle is a curve line, all the points of which are equally distant from a certain point within called the centre.

2. The circle is the space bounded by this curve line.

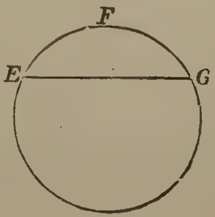
3. Every straight line, CA, CD, CE, drawn from the centre to the circumference, is called a *radius* or *semidiameter*. Every line which, like AB, passes through the centre and terminates in the circumference, is called a *diameter*.



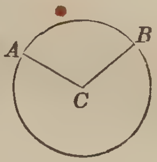
4. Any portion of the circumference, as EFG, is called an *arc*.

5. A straight line, as EG, joining the extremities of an arc, is called a *chord*.

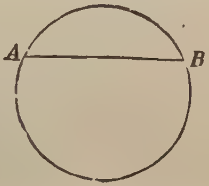
6 A *segment* is the surface or portion of a circle included between an arc and its chord. Thus EFG is a segment.



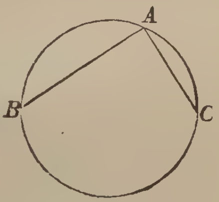
7. A *sector* is the part of the circle included between an arc and the two radii drawn through its extremities. Thus, *CAB* is a sector



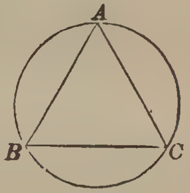
8. A straight line is said to be inscribed in a circle, when its extremities are in the circumference. Thus, the line *AB* is inscribed in a circle.



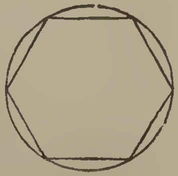
9. An inscribed angle is one which is formed by two chords that intersect each other in the circumference. Thus, *BAC* is an inscribed angle.



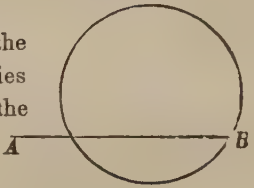
10. An inscribed triangle is one which has its three angular points in the circumference. Thus, *ABC* is an inscribed triangle.



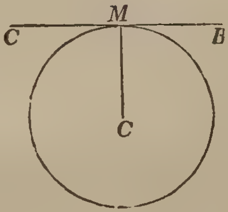
11. Any polygon is said to be inscribed in a circle when the vertices of all the angles are in the circumference. The circle is then said to circumscribe the polygon.



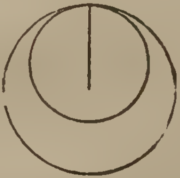
12 A *secant* is a line which meets the circumference in two points, and lies partly within and partly without the circle. Thus *AB* is a secant.



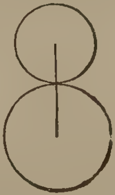
13. A *tangent* is a line which has but one point in common with the circumference. Thus, *CMB* is a tangent.



14. Two circles are said to touch each other internally, when one lies within the other, and their circumferences have but one point in common.



15. Two circles are said to touch each other externally, when one lies without the other, and their circumferences have but one point in common

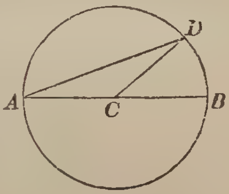


Of the Circle.

THEOREM I.

Every chord is less than a diameter.

Let AD be any chord. Draw the radii CA, CD to its extremities. We shall then have, AD less than $AC+CD$ (Book I. Th. x[*]). But $AC+CD$ is equal to the diameter AB: hence, the chord AD is less than the diameter.



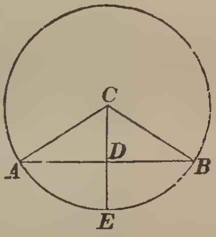
THEOREM II.

If from the centre of a circle a line be drawn to the middle of a chord,

I. *It will be perpendicular to the chord;*

II. *And it will bisect the arc of the chord.*

Let C be the centre of a circle, and AB any chord. Draw CD through D, the middle point of the chord, and produce it to E: then will CD be perpendicular to the chord, and the arc AE equal to EB.



First. Draw the two radii CA, CB. Then the two triangles ACD, DCB, have the three sides of the one equal to the three sides of the other, each to each: viz. AC equal to CB, being radii, AD equal to DB, by hypothesis, and CD common: hence, the corresponding angles are equal (Book I. Th. viii): that is, the angle CDA equal to CDB, and the angle ACD equal to the angle DCB.

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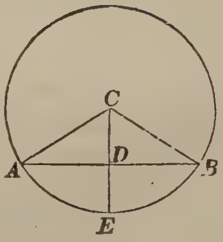
[*] *Note.* When reference is made from one theorem to another, in the same Book, the number of the theorem referred to is alone given: but when the theorem referred to is found in a preceding Book, the number of the Book is also given.



But, since the angle CDA is equal to the angle CDB, the radius CE is perpendicular to the chord AB (Bk. I. Def. 20).

Secondly. Since the angle ACE is equal to BCE, the arc AE will be equal to the arc EB, for equal angles must have equal measures (Bk. I. Def. 29).

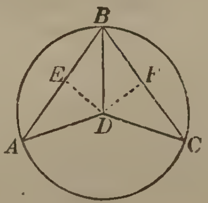
Hence, the radius drawn through the middle point of a chord, is perpendicular to the chord, and bisects the arc of the chord.

Cor. Hence, a line which bisects a chord at right angles, bisects the arc of the chord, and passes through the centre of the circle. Also, a line drawn through the centre of the circle and perpendicular to the chord, bisects it.

THEOREM III.

If more than two equal lines can be drawn from any point within a circle to the circumference, that point will be the centre.

Let D be any point within the circle ABC. Then, if the three lines DA, DB, and DC, drawn from the point D to the circumference, are equal, the point D will be the centre.



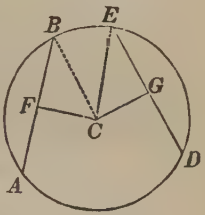
For, draw the chords AB, BC, bisect them at the points E and F, and join DE and DF.

Then, since the two triangles *DAE* and *DEB* have the side *AE* equal to *EB*, *AD* equal to *DB*, and *DE* common, they will be equal in all respects; and consequently, the angle *DEA* is equal to the angle *DEB* (Bk. I. Th. viii); and therefore, *DE* is perpendicular to *AB* (Bk. I. Def. 20) But if *DE* bisects *AB* at right angles, it will pass through the centre of the circle (Th. ii. Cor).

In like manner, it may be shown that *DF* passes through the centre of the circle, and since the centre is found in the two lines *ED*, *DF*, it will be found at their common intersection *D*.

THEOREM IV.

Any chords which are equally distant from the centre of a circle, are equal.



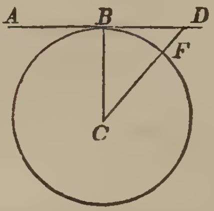
Let *AB* and *ED* be two chords equally distant from the centre *C*: then will the two chords *AB*, *ED* be equal to each other.

Draw *CF* perpendicular to *AB*, and *CG* perpendicular to *ED*, and since these perpendiculars measure the distances from the centre, they will be equal. Also draw *CB* and *CE*.

Then, the two right angled triangles *CFB* and *CEG* having the hypothenuse *CB* equal to the hypothenuse *CE*, and the side *CF* equal to *CG*, will have the third side *BF* equal to *EG* (Bk. I Th. xix) But, *BF* is the half of *BA* and *EG* the half of *DE* (Th. ii. Cor); hence *BA* is equal to *DE* (Ax 6).

THEOREM V.

A line which is perpendicular to a radius at its extremity, is tangent to the circle.



Let the line ABD be perpendicular to the radius CB at the extremity B: then will it be tangent to the circle at the point B.

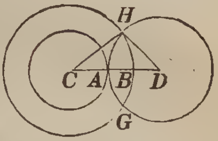
For, from any other point of the line, as D, draw DFC to the centre, cutting the circumference in F.

Then, because the angle B, of the triangle CDB, is a right angle, the angle at D is acute (Bk. I. Th. xvii. Cor. 3), and consequently less than the angle B. But the greater side of every triangle is opposite to the greater angle (Bk. I. Th. xi); therefore, the side CD is greater than CB, or its equal CF. Hence, the point D is without the circle, and the same may be shown for every other point of the line AD. Consequently, the line ABD has but one point in common with the circumference of the circle, and therefore is tangent to it at the point B (Def. 13)

Cor. Hence, if a line is tangent to a circle, and a radius be drawn through the point of contact, the radius will be perpendicular to the tangent.

THEOREM VI.

If the distance between the centres of two circles is equal to the sum of their radii, the two circles will touch each other externally.

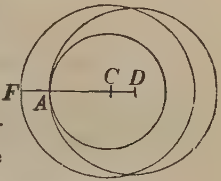


Let C and D be the two centres, and suppose the distance between them to be equal to the sum of the radii, that is, to $CA+AD$.

The circumferences of the circles will evidently have the point A common, and they will have no other. Because, if they had two points common, that is if they cut each other in two points, G and H, the distance CD between their centres would be less than the sum of their radii CH, HD (Bk. I. Th. x); but this would be contrary to the supposition.

THEOREM VII.

If the distance between the centres of two circles is equal to the difference of their radii, the two circles will touch each other internally.



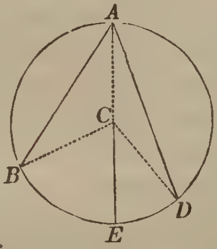
Let C and D be the centres of two circles at a distance from each other equal to $AD-AC=CD$.

Now, it is evident, as in the last theorem, that the circumferences will have the point A common; and they can have no other. For, if they had two points common, the difference between the radii AD and FC would not be equal to CD, the distance between their centres: therefore, they cannot have two points in common when the difference of their radii is equal to the distance between their centres: hence, they are tangent to each other.

Sch If two circles touch each other, either externally or internally, their centres and the point of contact will be in the same straight line

THEOREM VIII.

An angle at the circumference of a circle is measured by half the arc that subtends it



Let *BAD* be an inscribed angle: then will it be measured by half the arc *BED*, which subtends it.

For, through the centre *C* draw the diameter *ACE*, and draw the radii *BC*, *CD*.

Then, in the triangle *ABC*, the exterior angle *BCE* is equal to the sum of the interior angles *B* and *A* (Bk. I. Th. xvi). But since the triangle *BAC* is isosceles, the angles *A* and *B* are equal (Bk. I. Th. vi); therefore, the exterior angle *BCE* is equal to double the angle *BAC*.

But, the angle *BCE* is measured by the arc *BE*, which subtends it; and consequently, the angle *BAE*, which is half of *BCE*, is measured by half the arc *BE*.

It may be shown, in like manner, that the angle *EAD* is measured by half the arc *ED:* and hence, by the addition of equals, it would follow that, the angle *BAD* is measured by half the arc *BED*, which subtends it.

Cor. 1. Hence, if an angle at the centre, and an angle at the circumference, both stand on the same arc, the angle at the centre will be double the angle at the circumference.

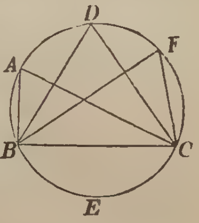
Cor. 2. If two angles at the circumference stand on equal arcs they will be equal to each other.

THEOREM IX.

All angles at the circumference, which stand upon the same arc are equal to each other.

Let the angles *BAC*, *BDC*, *BFC*, have their vertices in the circumference, and stand on the same arc *BEC*: then will they be equal to each other.

For, each angle is measured by half the arc *BEC* (Th. viii); hence, the angles are all equal.

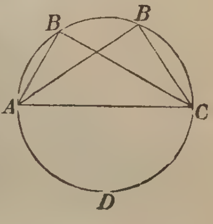


THEOREM X.

An angle in a semicircle, is a right angle.

Let *ABBC* be a semicircle: then will every angle, as *B*, *B*, inscribed in it, be a right angle.

For, each angle is measured by half the semicircumference *ADC*, that is, by a quadrant, which measures a right angle (Bk I. Th. i. Cor. 2).

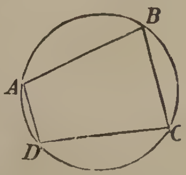


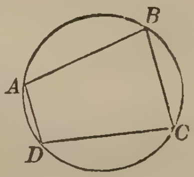
THEOREM XI.

If a quadrilateral be inscribed in a circle, the sum of either two of its opposite angles is equal to two right angles.

Let *ABCD* be any quadrilateral inscribed in a circle; then will the sum of the two opposite angles, *A* and *C*, or *B* and *D*, be equal to two right angles.

For, the angle *A* is measured by half the arc *DCB*, which subtends it (Th. viii);



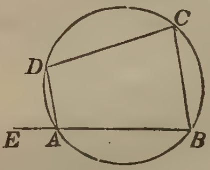


and the angle C is measured by half the arc DAB, which subtends it. Hence, the sum of the two angles, A and C, is measured by half the entire circumference. But half the entire circumference is the measure of two right angles; therefore, the sum of the opposite angles A and C is equal to two right angles.

In like manner, it may be shown, that the sum of the two angles B and D is equal to two right angles

THEOREM XII.

If the side of a quadrilateral, inscribed in a circle, be produced out, the exterior angle will be equal to the inward opposite angle



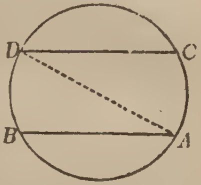
Let the side BA, of the quadrilateral $ABCD$ be produced to E, then will the outward angle DAE be equal to the inward opposite angle C.

For, the angle DAB plus the angle C, is equal to two right angles (Th. xi). But DAB plus DAE is also equal to two right angles (Bk. I. Th. ii). Taking from each the common angle DAB, and we shall have the angle DAE equal to the interior opposite angle C.

THEOREM XIII.

Two parallel chords intercept equal arcs.

Let the chords *AB* and *CD* be parallel: then will the arcs *AC* and *BD* be equal.

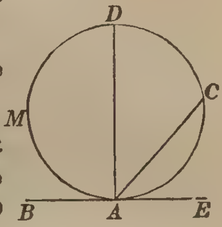


For, draw the line *AD*. Then, because the lines *AB* and *CD* are parallel, the alternate angles *ADC* and *DAB* will be equal (Bk. I. Th. xii). But the angle *ADC* is measured by half the arc *AC*, and the angle *DAB* by half the arc *BD* (Th. viii): hence the two arcs *AC* and *BD* are themselves equal.

THEOREM XIV.

The angle formed by a tangent and a chord, is measured by half the arc of the chord.

Let *BAE* be tangent to the circle at the point *A*, and *AC* any chord.



From *A*, the point of contact, draw the diameter *AD*.

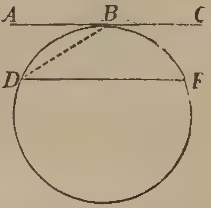
Then, the angle *BAD* will be a right angle (Th. v. Cor), and therefore will be measured by half the semicircle *AMD* (Bk. I, Th. i. Cor. 2).

But the angle *DAC* being at the circumference, is measured by half the arc *DC*: hence, by the addition of equals, the two angles *BAD* and *DAC*, or the entire angle *BAC* will be measured by half the arc *AMDC*.

It may be shown, by taking the difference between the two angles *DAE* and *DAC*, that the angle *CAE* is measured by half the arc *AC* included between its sides.

THEOREM XV.

If a tangent and a chord are parallel to each other, they will intercept equal arcs.

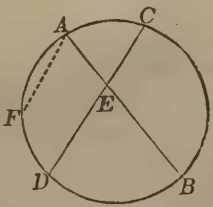


Let the tangent ABC be parallel to the chord DF: then will the intercepted arcs DB, BF, be equal to each other.

For, draw the chord DB. Then, since AC and DF are parallel, the angle ABD will be equal to the angle BDF. But ABD being formed by a tangent and a chord, will be measured by half the arc DB; and BDF being an angle at the circumference will be measured by half the arc BF (Th. viii). But since the angles are equal, the arcs will be equal: hence DB is equal to BF.

THEOREM XVI.

The angle formed within a circle by the intersection of two chords, is measured by half the sum of the intercepted arcs.



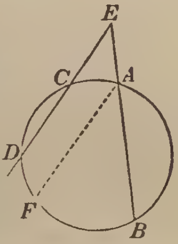
Let the two chords AB and CD intersect each other at the point E: then will the angle AEC, or its equal DEB, be measured by half the sum of the intercepted arcs AC, DB.

For, draw the chord AF parallel to CD. Then because of the parallels, the angle DEB will be equal to the angle FAB (Bk I. Th. xiv), and the arc FD to the arc AC. But the angle FAB is measured by half the arc FDB, that is, by half the sum of the arcs FD, DB. Now, since FD is equal to AC, it follows that the angle DEB, or its equal AEC, will be measured by half the sum of the arcs DB and AC

THEOREM XVII.

The angle formed without a circle by the intersection of two secants is measured by half the difference of the intercepted arcs

Let the two secants *DE* and *EB* intersect each other at *E*: then will the angle *DEB* be measured by half the intercepted arcs *CA* and *DB*.



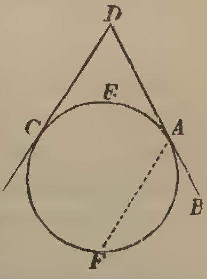
Draw the chord *AF* parallel to *ED*. Then, because *AF* and *ED* are parallel, and *EB* cuts them, the angles *FAB* and and *DEB* are equal (Bk. I. Th. xiv).

But the angle *FAB*. at the circumference, is measured by half the arc *FB* (Th. viii), which is the difference of the arcs *DFB* and *CA*: hence, the equal angle *E* is also measured by half the difference of the intercepted arcs *DFB* and *CA*

THEOREM XVIII.

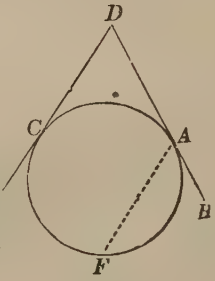
An angle formed by two tangents is measured by half the difference of the intercepted arcs.

Let *CD* and *DA* be two tangents to the circle at the points *C* and *A*: then will the angle *CDA* be measured by half the difference of the intercepted arcs *CEA* and *CFA*.

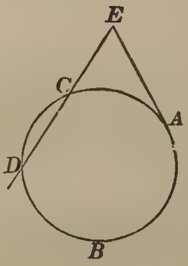


For, draw the chord *AF* parallel to the tangent *CD*. Then, because the lines *CD* and *AF* are parallel, the angle *BAF* will be equal to the angle *BDC* (Bk. I. Th. xiv). But the angle *BAF*, formed by a tangent and a chord, is measured by half the arc AF, that is, by half the difference of CFA and CF.

But since the tangent DC and the chord AF are parallel, the arc CF is equal to the arc CA: hence the angle BAF, or its equal BDC, which is measured by half the difference of CFA and CF, is also measured by half the difference of the intercepted arcs CFA and CA.



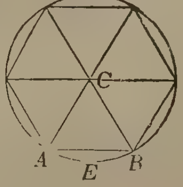
Cor. In like manner it may be proved that the angle E, formed by a tangent and secant, is measured by half the difference of the intercepted arcs AC and DBA.



THEOREM XIX.

The chord of an arc of sixty degrees is equal to the radius of the circle.

Let AEB be an arc of sixty degrees and AB its chord: then will AB be equal to the radius of the circle.



For, draw the radii CB and CA. Then, since the angle ACB is at the centre, it will be measured by the arc AEB: that is, it will be equal to sixty degrees (Bk. I. Def. 29).

Again, since the sum of the three angles of a triangle is equal to one hundred and eighty degrees (Bk. I. Th. xvii), it follows that the sum of the two angles A and B will be equal to one hundred and twenty degrees. But the triangle CAB is isosceles: hence, the angles at the base are equal (Bk. I. Th. vi): hence, each angle is equal to sixty degrees, and consequently, the side AB is equal to AC or CB (Bk. I. Th. vi).

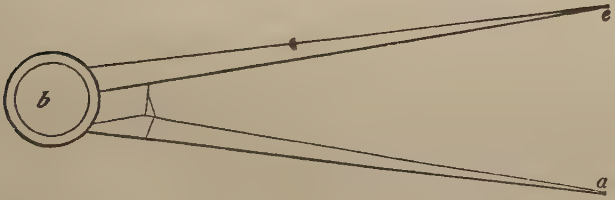
PROBLEMS

RELATING TO THE FIRST AND SECOND BOOKS.

The Problems of Geometry explain the methods of constructing or describing the geometrical figures.

For these constructions, a straight ruler and the common compasses or dividers, are all the instruments that are absolutely necessary.

DIVIDERS OR COMPASSES.

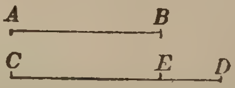


The dividers consist of the two legs ba, be, which turn easily about a common joint at b. The legs of the dividers are extended or brought together by placing the forefinger on the joint at *b*, and pressing the thumb and fingers against the legs.

PROBLEM I.

On any line, as CD, to lay off a distance equal to AB.

Take up the dividers with the thumb and second finger, and place the forefinger on the joint at *b*.



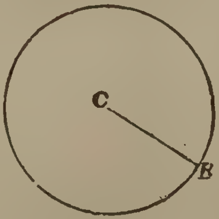
Then, set one foot of the dividers at *A*, and extend the legs with the thumb and fingers, until the other foot reaches *B*.

Then, raise the dividers, place one foot at *C*, and mark with the other the distance *CE*: and this distance will evidently be equal to *AB*.

PROBLEM II.

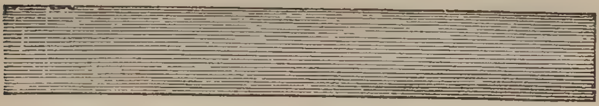
To describe from a given centre the circumference of a circle having a given radius.

Let *C* be the given centre, and *CB* the given radius.



Place one foot of the dividers at *C* and extend the other leg until it reaches to B. Then, turn the dividers around the leg at *C*, and the other leg will describe the required circumference.

OF THE RULER.



A ruler of a convenient size, is about twenty inches in length, two inches wide, and one fifth of an inch in thickness. It should be made of a hard material, and perfectly straight and smooth.

PROBLEM III.

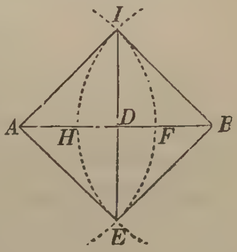
To draw a straight line through two given points A and B.

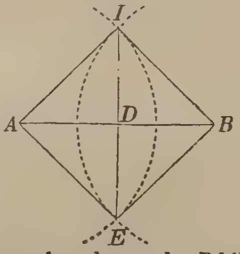
Place one edge of the ruler on *A* and slide the ruler around until the same edge falls on *B*. Then, with a pen, or pencil, draw the line *AB*.

PROBLEM IV.

To bisect a given line: that is, to divide it into two equal parts.

Let *AB* be the given line to be divided. With *A* as a centre, and radius greater than half of *AB*, describe an arc *IFE*. Then, with *B* as a centre, and an equal radius *BI* describe the arc *IHE*. Join the points *I* and *E* by the line *IE*: the point *D*, where it intersects *AB*, will be the middle point of the line *AB*.

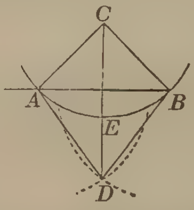




For, draw the radii AI, AE BI, and BE. Then, since these radii are equal, the triangles AIE and BIE have all the sides of the one equal to the corresponding sides of the other; hence, their corresponding angles are equal (Bk. I. Th. viii); that is, the angle AIE is equal to the angle BIE. Therefore, the two triangles AID and BID, have the side $AI=IB$, the angle $AID=BID$, and ID common: hence they are equal (Bk. I. Th. iv), and AD is equal to DB.

PROBLEM V.

To bisect a given angle or a given arc.



Let ACB be the given angle, and AEB the given arc.

From the points A and B, as centres, describe with the same radius two arcs cutting each other in D. Through D and the centre C, draw CED, and it will divide the angle ACB into two equal parts, and also bisect the arc AEB at E.

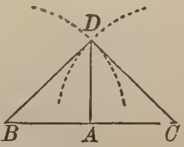
For, draw the radii AD and BD. Then, in the two triangles ACD, CBD, we have

$$AC=CB, \qquad AD=BD$$

and CD common: hence, the two triangles have their corresponding angles equal (Bk I. Th. viii), and consequently, ACD is equal to BCD. But since ACD is equal to BCD, it follows that the arc AE, which measures the former, is equal to the arc BE, which measures the latter

PROBLEM VI.

At a given point in a straight line to erect a perpendicular to the line.



Let A be the given point, and BC the given line.

From A lay off any two distances, AB and AC, equal to each other Then, from the points B and C, as centres, with a radius greater than AB, describe two arcs intersecting each other at D; draw DA, and it will be the perpendicular required.

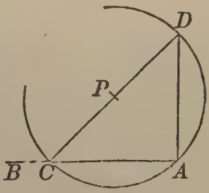
For, draw the equal radii BD, DC. Then, the two triangles, BDA, and CDA, will have

$$AB=AC \qquad BD=DC$$

and AD common: hence, the angle DAB is equal to the angle DAC (Bk. I. Th. viii), and consequently, DA is perpendicular to BC. (Bk. I Def. 21).

SECOND METHOD.

When the point A is near the extremity of the line.

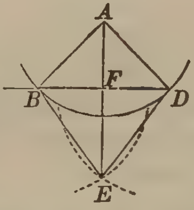


Assume any centre, as P, out of the given line. Then with P as a centre, and radius from P to A, describe the circumference of a circle Through C, where the circumference cuts BA, draw CPD. Then, through D, where CP produced meets the circumference, draw DA: then will DA be perpendicular to BA, since CAD is an angle in a semicircle (Bk. II. Th. x).

PROBLEM VII.

From a given point without a straight line to let fall a perpendicular on the line.

Let A be the given point, and BD the given line

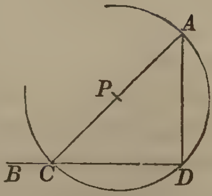


From the point A as a centre, with a radius greater than the shortest distance to BD, describe an arc cutting BD in the points B and D. Then, with B and D as centres, and the same radius, describe two arcs intersecting each other at E. Draw AFE, and it will be the perpendicular required.

For, draw the equal radii AB, AD, BE and DE. Then, the two triangles EAB and EAD will have the sides of the one equal to the sides of the other, each to each; hence, their corresponding angles will be equal (Bk. I. Th. viii), viz. the angle BAE to the angle DAE. Hence, the two triangles BAF and DAF will have two sides and the included angle of the one, equal to two sides and the included angle of the other, and therefore, the angle AFB will be equal to the angle AFD (Bk. I. Th. iv): hence, AFE will be perpendicular to BD.

SECOND METHOD.

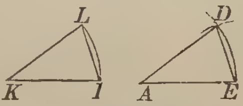
When the given point A is nearly opposite the extremity of the line.



Draw AC, to any point C of the line BD. Bisect AC at P. Then, with P as a centre and PC as a radius, describe the semicircle CDA; draw AD, and it will be perpendicular to CD, since CDA is an angle in a semicircle (Bk. II. Th. x).

PROBLEM VIII.

At a given point in a given line, to make an angle equal to a given angle.



Let A be the given point, AE the given line, and IKL the given angle.

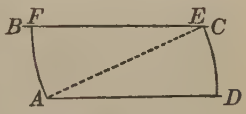
From the vertex K, as a centre, with any radius, describe the arc IL, terminating in the two sides of the angle: and draw the chord IL.

From the point A, as a centre, with a distance AE, equal to KI, describe the arc DE; then with E, as a centre, and a radius equal to the chord IL, describe an arc cutting DE at D; draw AD, and the angle EAD will be equal to the angle K.

For, draw the chord DE. Then the two triangles IKL and EAD, having the three sides of the one equal to the three sides of the other, each to each, the angle EAD will be equal to the angle K (Bk. I. Th. viii).

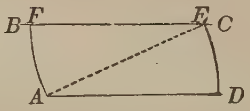
PROBLEM IX.

Through a given point to draw a line that shall be parallel to a given line.



Let A be the given point and BC the given line.

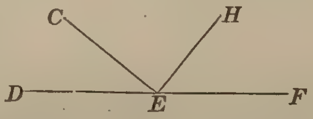
With A as a centre, and any radius greater than the shortest distance from A to BC, describe the indefinite arc DE. From the point E, as a centre, with the same radius, describe the arc AF: then, make ED equa to AF and draw AD, and it will be the required parallel.



For, since the arcs AF and ED are equal, the angles EAD and AEF, which they measure, are equal: hence, the line AD is parallel to BC (Bk I. Th xiii).

PROBLEM X.

Two angles of a triangle being given or known, to find the third.



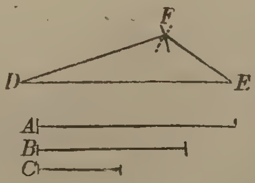
Draw the indefinite line DEF.

At any point, as E, make the angle DEC equal to one of the given angles, and then CEH equal to a second, by Prob. VIII; then will the angle HEF be equal to the third angle of the triangle.

For, the sum of the three angles of a triangle is equal to two right angles (Bk. I. Th. xvii); and the sum of the three angles on the same side of the line DE is equal to two right angles (Bk. I. Th. ii. Cor. 2); hence, if DEC and CEH are equal to two of the angles, the angle HEF will be equal to the remaining angle of the triangle

PROBLEM XI.

Three sides of a triangle being given, to describe the triangle.



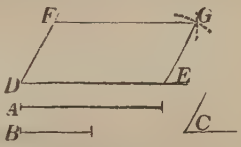
Let A, B, and C, be the given sides.

Draw DE, and make it equal to the side A. From the point D, as a centre, with a radius equal to the second side B, describe an arc from *E* as a centre, with the third side *C*, describe another arc intersecting the former in *F:* draw *DF* and *FE:* then will *DEF* be the required triangle.

For, the three sides are respectively equal to the three lines *A*, *B*, and *C*.

PROBLEM XII.

The adjacent sides of a parallelogram, with the angle which they contain, being given, to describe the parallelogram



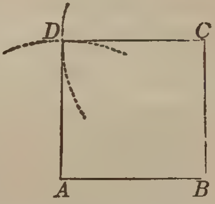
Let *A* and *B* be the given sides and *C* the given angle.

Draw the line *DE* and make it equal to *A*. At the point *D* make the angle *EDF* equal to the angle *C*. Make the side *DF* equal to *B*. Then describe two arcs, one from *F* as a centre, with a radius *FG* equal to *DE*, the other from *E*, as a centre, with a radius *EG* equal to *DF*. Through the point *G*, the point of intersection, draw the lines *EG* and *FG*, and *DEGF* will be the required parallelogram.

For, in the quadrilateral *DFGE*, the opposite sides *DE* and *FG* are each equal to *A:* the opposite sides *DF* and *EG* are each equal to *B*, and the angle *EDF* is equal to *C*. But, since the opposite sides are equal, they are also parallel (Bk. I Th. xxiv), and therefore the figure is a parallelogram

PROBLEM XIII.

To describe a square on a given line.



Let AB be the given line.

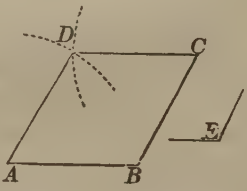
At the point B draw BC perpendicular to AB, by Problem VI, and then make it equal to AB.

Then, with A as a centre, and radius equal to AB, describe an arc; and with C as a centre, and the same radius AB, describe another arc; and through D, their point of intersection, draw AD and CD: then will $ABCD$ be the required square.

For, since the opposite sides are equal, the figure will be a parallelogram (Bk. I. Th. xxiv): and since one of the angles is a right angle, the others will also be right angles (Bk. I. Th. xxiii. Cor. 1); and since the sides are all equal, the figure will be a square.

PROBLEM XIV.

To construct a rhombus, having given the length of one of the equal sides, and one of the angles.



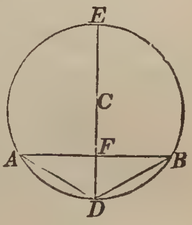
Let AB be equal to the given side, and E the given angle.

At B lay off an angle, ABC, equal to E, by Prob. VIII. and make BC equal to AB. Then, with A and C as centres, and a radius equal to AB, describe two arcs. Through D, their point of intersection, draw the lines AD, CD: then will $ABCD$ be the required rhombus.

For, since the opposite sides are equal, they will be parallel (Bk. 1. Th. xxiv). But they are each equal to AB, and the angle B is equal to the angle E: hence, $ABCD$ is the required rhombus.

PROBLEM XV.

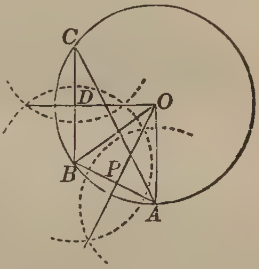
To find the centre of a circle



Draw any chord, as AB, and bisect it by Problem IV. Then, through F, the middle point, draw DCE, perpendicular to AB, by Problem VI. Then DCE will be a diameter of the circle (Bk. II. Th. ii. Cor.). Then bisect DE at C, and C will be the centre of the circle.

PROBLEM XVI.

To describe the circumference of a circle through three given points not in the same straight line.

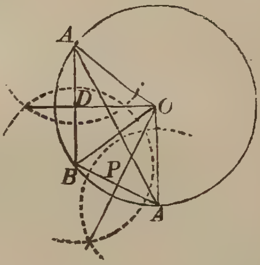


Let A, B, C, be the given points. Join these points by the straight lines AC AB, BC.

Then, bisect any two of these straight lines, as AB, BC, by the perpendiculars OD, OP (Prob. iv); and the point O, where these perpendiculars intersect each other, will be the centre of the circle.

Then with O as a centre, and a radius equal to OA, describe the circumference of a circle, and it will pass through the points A, B, and C.

For, the two right angled triangles OAP and OBP have the side AP equal to the side BP, OP common, and the included



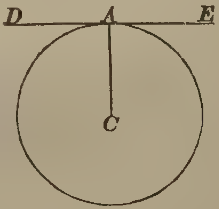
angles OPA and OPB equal, being right angles; hence, the side OB is equal to OA (Bk. I. Th. iv).

In like manner it may be shown that OC is equal to OB. Hence, a circumference described with the radius OA, will pass through the points B and C.

Sch. This problem enables us to describe the circumference of a circle about a given triangle. For, we may consider the vertices of the three angles as the three points through which the circumference is to pass.

PROBLEM XVII.

Through a given point in the circumference of a circle, to draw a tangent line to the circle.

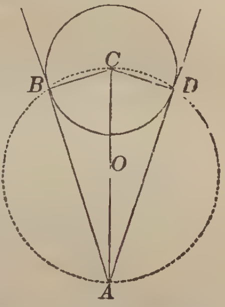


Let A be the given point

Through A, draw the radius AC to the centre, and then draw DAE perpendicular to AC, by Problem VI. Then will DAE be tangent to the circle at the point A (Bk. II. Th. v).

PROBLEM XVIII.

Through a given point without the circumference, to draw a tangent line to the circle.



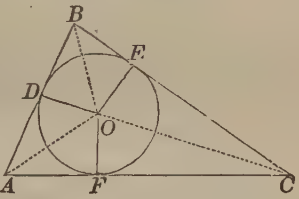
Let C be the centre of the circle, and A the given point without the circle.

Join A and the centre C, and on AC, as a diameter, describe a circumference. Through the points B and D where the two circumferences intersect each other, draw the lines AB and AD: these lines will be tangent to the circle whose centre is C.

For, since the angles ABC and ADC are each inscribed in a semicircle, they will be right angles (Bk. II. Th. x). Again, since the lines AB, AD, are each perpendicular to a radius at its extremity, they will be tangent to the circle (Bk. II. Th. v).

PROBLEM XIX

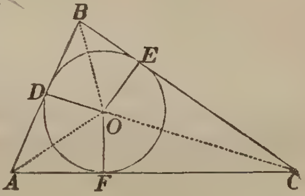
To inscribe a circle in a given triangle.



Let ABC be the given triangle.

Bisect the angles A and B by the lines AO and BO, meeting at the point O. From O, let fall the perpendiculars OD, OE, OF, on the three sides of the triangle—these perpendiculars will be equal to each other.

For, in the two right angled triangles DAO and FAO, we have the right angle D equal the right angle F, the angle FAO equal to DAO, and consequently, the third angles AOD and AOF are equal (Bk. I. Th. xvii. Cor 1) But the two triangles have a common side AO, hence, they are equal (Bk. I. Th. v), and consequently, OD is equal to OF

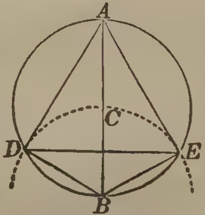


In a similar manner, it may be proved that OE and OD are equal: hence, the three perpendiculars, OD, OF, and OE, are all equal.

Now, if with O as a centre, and OF as a radius, we describe the circumference of a circle, it will pass through the points D and E. and since the sides of the triangle are perpendicular to the radii OF, OD, OE, they will be tangent to the circumference (Bk. II. Th. v). Hence, the circle will be inscribed in the triangle.

PROBLEM XX.

To inscribe an equilateral triangle in a circle.



Through the centre C draw any diameter, as ACB. From B as a centre, with a radius equal to BC, describe the arc DCE. Then, draw AD, AE, and DE, and DAE will be the required triangle.

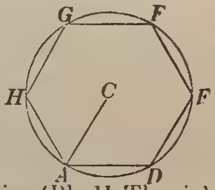
For, since the chords BD, BE, are each equal to the radius CB, the arcs BD, BE, are each equal to sixty degrees (Bk. II. Th. xix), and the arc DBE to one hundred and twenty degrees; hence, the angle DAE is equal to sixty degrees (Bk. II. Th. viii).

Again, since the arc BD is equal to sixty degrees, and the arc BDA equal to one hundred and eighty degrees, it follows that DA will be equal to one hundred and twenty degrees: hence, the angle DEA is equal to sixty degrees, and consequently, the third angle ADE, is equal to sixty degrees

Therefore, the triangle *ADE* is equilateral (Bk. I. Th. vi Cor. 2).

PROBLEM XXI.

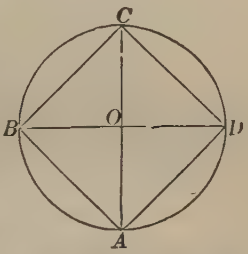
To inscribe a regular hexagon in a circle.



Draw any radius, as *AC*. Then apply the radius *AC* around the circumference, and it will give the chords *AD*, *DE*, *EF*, *FG*, *GH*, and *HA*, which will be the sides of the regular hexagon. For, the side of a hexagon is equal to the radius (Bk. II. Th. xix).

PROBLEM XXII.

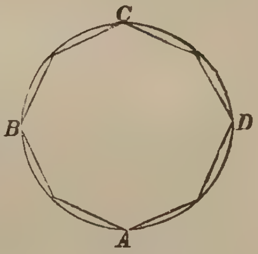
To inscribe a square in a given circle.



Let *ABCD* be the given circle. Draw the two diameters *AC*, *BD*, at right angles to each other, and through the points *A*, *B*, *C* and *D* draw the lines *AB*, *BC*, *CD*, and *DA*: then will *ABCD* be the required square.

For, the four right angled triangles, *AOB*, *BOC*, *COD*, and *DOA* are equal, since the sides *AO*, *OB*, *OC*, and *OD* are equal, being radii of the circle; and the angles at *O* are equal in each, being right angles: hence, the sides *AB*, *BC*, *CD*, and *DA* are equal (Bk. I. Th. iv).

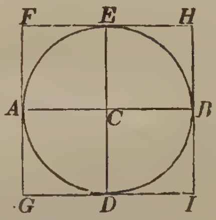
But each of the angles *ABC*, *BCD*, *CDA*, *DAB*, is a right angle, being an angle in a semicircle (Bk. II. Th x): hence, the figure *ABCD* is a square (Bk. I. Def. 48)



Sch. If we bisect the arcs *AB*, *BC*, *CD*, *DA*, and join the points, we shall have a regular octagon inscribed in the circle. If we again bisect the arcs, and join the points of bisection, we shall have a regular polygon of sixteen sides.

PROBLEM XXIII.

To describe a square about a given circle.



Draw the diameters *AB*, *DE*, at right angles to each other. Through the extremities *A* and *B* draw *FAG* and *HBI* parallel to *DE*, and through *E* and *D*, draw *FEH* and *GDI* parallel to *AB*: then will *FGIH* be the required square.

For, since *ACDG* is a parallelogram, the opposite sides are equal (Bk. I. Th. xxiii): and since the angle at *C* is a right angle all the other angles are right angles (Bk. I. Th. xxiii. Cor. 1): and as the same may be proved of each of the figures *CI*, *CH* and *CF*, it follows that all the angles, *F*, *G*, *I*, and *H*, are right angles, and that the sides *GI*, *IH*, *HF*, and *FG*, are equal, each being equal to the diameter of the circle. Hence the figure *GIHF* is a square (Bk I. Def. 48).

GEOMETRY.

BOOK III.

OF RATIOS AND PROPORTIONS.

DEFINITIONS.

1. *Ratio* is the quotient arising from dividing one quantity by another quantity of the same kind. Thus, if the numbers 3 and 6 have the same unit, the ratio of 3 to 6 will be expressed by

$$\frac{6}{3}=2$$

And in general, if A and B represent quantities of the same kind, the ratio of A to B will be expressed by

$$\frac{B}{A}.$$

2. If there be four numbers, 2, 4, 8, 16, having such values that the second divided by the first is equal to the fourth divided by the third, the numbers are said to be in proportion. And in general, if there be four quantities A, B, C, and D having such values that

$$\frac{B}{A}=\frac{D}{C},$$

then, A is said to have the same *ratio* to B, that C has to D, or, the ratio of A to B is equal to the ratio of C to D When four quantities have this relation to each other, they are said to be in proportion. Hence, the proportion of four quantities results from an equality of their ratios taken two and two.

To express that the ratio of A to B is equal to the ratio of C to D, we write the quantities thus :

$$A : B :: C : D;$$

and read, A is to B, as C to D.

The quantities which are compared together are called the *terms* of the proportion. The first and last terms are called the *extremes*, and the second and third terms, the *means*. Thus, A and D are the extremes, and B and C the means.

3. Of four proportional quantities, the first and third are called the *antecedents*, and the second and fourth the *consequents ;* and the last is said to be a fourth proportional to the other three taken in order. Thus, in the last proportion, A and C are the antecedents, and B and D the consequents.

4. Three quantities are in proportion when the first has the same ratio to the second, that the second has to the third ; and then the middle term is said to be a mean proportional between the two other. For example,

$$3 : 6 :: 6 : 12;$$

and 6 is a mean proportional between 3 and 12.

5. Quantities are said to be in proportion by *inversion*, or *inversely*, when the consequents are made the antecedents and the antecedents the consequents.

Thus, if we have the proportion

$$3 : 6 :: 8 : 16.$$

the inverse proportion would be

$$6 : 3 :: 16 : 8.$$

6 Quantities are said to be in proportion by *alternation*, or *alternately*, when antecedent is compared with antecedent and consequent with consequent.

Thus, if we have the proportion

$$3 : 6 :: 8 : 16,$$

the alternate proportion would be

$$3 : 8 :: 6 : 16.$$

7. Quantities are said to be in proportion by *composition*, when the sum of the antecedent and consequent is compared either with antecedent or consequent.

Thus, if we have the proportion

$$2 : 4 :: 8 : 16,$$

the proportion by composition would be

$$2+4 : 4 :: 8+16 : 16;$$

that is, $6 : 4 :: 24 : 16.$

8. Quantities are said to be in proportion by *division*, when the difference of the antecedent and consequent is compared either with the antecedent or consequent.

Thus, if we have the proportion

$$3 : 9 :: 12 : 36,$$

the proportion by division will be

$$9-3 : 9 :: 36-12 : 36;$$

that is, $6 : 9 :: 24 : 36.$

9. Equimultiples of two or more quantities are the products which arise from multiplying the quantities by the same number.

Thus, if we have any two numbers, as 6 and 5 and multiply them both by any number, as 9, the equimultiples will be 54 and 45; for

$$6\times9=54 \quad \text{and} \quad 5\times9=45.$$

Also, $m\times A$ and $m\times B$ are equimultiples of A and B, the common multiplier being m.

10. Two variable quantities, A and B, are said to be *reciprocally proportional*, or *inversely proportional*, when one increases in the same ratio as the other diminishes. When this relation exists, either of them is equal to a constant quantity divided by the other.

Thus, if we had any two numbers, as 2 and 4, so related to each other that if we divided one by any number we must multiply the other by the same number, one would increase in the same ratio as the other would diminish, and their product would not be changed.

THEOREM I.

If four quantities are in proportion, the product of the two extremes will be equal to the product of the two means.

If we have the proportion

$$A : B :: C : D$$

we have, by Def. 2,

$$\frac{B}{A}=\frac{D}{C}$$

and by clearing the equation of fractions, we have

$$BC=AD$$

Sch. The general principle is verified in the proportion between the numbers

$$2 : 10 :: 12 : 60$$

which gives

$$2\times60=10\times12=120$$

THEOREM II.

If four quantities are so related to each other, that the product of two of them is equal to the product of the other two; then two of them may be made the means, and the other two the extremes of a proportion.

Let A, B, C, and D, have such values that

$$B\times C=A\times D$$

Divide both sides of the equation by A and we have

$$\frac{B}{A}\times C=D$$

Then divide both sides of the last equation by C, and we have

$$\frac{B}{A}=\frac{D}{C}$$

hence, by Def. 2, we have

$$A \;:\; B \;::\; C \;:\; D.$$

Sch. The general truth may be verified by the numbers

$$2\times 18=9\times 4$$

which give

$$2 \;:\; 4 \;::\; 9 \;:\; 18$$

THEOREM III.

If three quantities are in proportion, the product of the two extremes will be equal to the square of the middle term.

Let us suppose that we have

$$A \;:\; B \;::\; B \;:\; C$$

Then, by Def. 2, we have

$$\frac{B}{A}=\frac{C}{B}$$

and by clearing the equation of its fractions, we have

$$B^2 = C \times A$$

Sch. The proposition may be verified by the numbers

$$3 : 6 :: 6 : 12$$

which give

$$3 \times 12 = 6 \times 6 = 36$$

THEOREM IV.

If four quantities are in proportion, they will be in proportion when taken alternately.

Let $A : B :: C : D$

Then, by Def. 2, we have

$$\frac{B}{A} = \frac{D}{C}$$

Multiplying both members of this equation by $\frac{C}{B}$, we have

$$\frac{C}{A} = \frac{D}{B}$$

and consequently,

$$A : C :: B : D.$$

Sch. The theorem may be verified by the proportion

$$10 : 15 :: 20 : 30$$

for, we have, by alternation,

$$10 : 20 :: 15 : 30.$$

THEOREM V.

If there be two sets of proportions, having an antecedent and a consequent in the one, equal to an antecedent and a consequent in the other; then, the remaining terms will be proportional.

If we have

$A : B :: C : D$, and $A : B :: E : F$;

then we shall have

$$\frac{B}{A}=\frac{D}{C} \quad \text{and} \quad \frac{B}{A}=\frac{F}{E}$$

Hence, by Ax. 1, we have

$$\frac{D}{C}=\frac{F}{E}$$

and consequently,

$$C : D :: E : F.$$

Sch. The proposition may be verified by the following proportions,

2 : 6 :: 8 : 24 and 2 : 6 :: 10 : 30

which give

8 : 24 :: 10 : 30.

THEOREM VI.

If four quantities are in proportion, they will be in proportion when taken inversely.

If we have the proportion

$$A : B :: C : D$$

we have, by Th. I,

$$A\times D=B\times C,$$

or

$$B\times C=A\times D.$$

Hence, we have, by Th. II,

$$B : A :: D : C.$$

Sch. The proposition may be verified by the proportion

7 : 14 :: 8 : 16;

which, when taken inversely, gives

14 : 7 :: 16 : 8.

THEOREM VII.

If four quantities are in proportion, they will be in proportion by composition.

Let us suppose that we have

$$A : B :: C : D$$

we shall then have

$$A \times D = B \times C.$$

To each of these equals, add $B \times D$, and we have

$$(A+B) \times D = (C+D) \times B;$$

and by separating the factors by Th. II, we have

$$A+B : B :: C+D : D.$$

Sch. The proposition may be verified by the following proportion,

$$9 : 27 :: 16 : 48.$$

We shall have, by composition,

$$9+27 : 27 :: 16+48 : 48,$$

that is, $36 : 27 :: 64 : 48$

in which the ratio is three fourths.

THEOREM VIII.

If four quantities are in proportion, they will be in proportion by division.

Let us suppose that we have

$$A : B :: C : D;$$

we shall then have

$$A \times D = B \times C.$$

From each of these equals let us subtract $B \times D$, and we have

$$(A-B) \times D = (C-D) \times B;$$

and by separating the factors by Th. II, we have,

$$A-B : B :: C-D : D.$$

Sch The proposition may be verified by the proportion,

$$24 \cdot 8 :: 48 : 16.$$

∴ We have, by division,

$$24-8 : 8 :: 48-16 : 16;$$

that is, $16 : 8 :: 32 : 16;$

in which the ratio is one-half.

THEOREM IX.

Equal multiples of two quantities have the same ratio as the quantities themselves.

If we have the proportion

$$A : B :: C : D$$

we shall have

$$\frac{B}{A}=\frac{D}{C}$$

Now, let M be any number, and by it multiply the numerator and denominator of the first member of the equation which will not change its value: we shall then have

$$\frac{M\times B}{M\times A}=\frac{D}{C}$$

and hence we have

$$M\times A : M\times B :: C : D,$$

that is, the equal multiples $M\times A$ and $M\times B$, have the same ratio as A to B.

Sch. The proposition may be verified by the proportion,

$$5 : 10 :: 12 : 24;$$

for, by multiplying the first antecedent and consequent by any number, as 6, we have

$$30 : 60 :: 12 : 24,$$

in which the ratio is still 2.

THEOREM X.

If four quantities are proportional, and one antecedent and its consequent be augmented by quantities which have the same ratio as the antecedent and consequent, the four quantities will still be in proportion.

Let us take the proportions

$A : B :: C : D$, and $A : B :: E : F$, which give

$$A \times D = B \times C \qquad \text{and} \qquad A \times F = B \times E;$$

adding these equals we have

$$A \times (D+F) = B \times (C+E);$$

and by Th. II, we have

$$A : B :: C+E : D+F$$

in which the antecedent C and its consequent D, are augmented by the quantities E and F, which have the same ratio.

Sch. The proposition may be verified by the proportion,

$$9 : 18 :: 20 : 40,$$

in which the ratio is 2.

If we augment the antecedent and its consequent by 15 and 30, which have the same ratio, we have

$$9 : 18 :: 20+15 : 40+30$$

that is, $9 : 18 :: 35 : 70,$

in which the ratio is still 2.

THEOREM XI.

If four quantities are proportional, and one antecedent and its consequent be diminished by quantities which have the same ratio as the antecedent and consequent, the four quantities will still be in proportion

Let us take the proportions

$A : B :: C : D$, and $A : B :: E : F$.
which give

$$A \times D = B \times C \quad \text{and} \quad A \times F = B \times E.$$

By subtracting these equalities, we have

$$A \times (D-F) = B \times (C-E);$$

and by Th. II, we obtain

$$A : B :: C-E : D-F,$$

in which the antecedent and consequent, C and D, are diminished by E and F, which have the same ratio.

Sch. The proposition may be verified by the proportion,

$$9 : 18 :: 20 : 40,$$

for, by diminishing the antecedent and consequent by 15 and 30, we have

$$9 : 18 :: 20-15 : 40-30;$$

that is $9 : 18 :: 5 : 10$

in which the ratio is still 2.

THEOREM XII.

If we have several sets of proportions, having the same ratio, any antecedent will be to its consequent, as the sum of the antecedents to the sum of the consequents.

If we have the several proportions,

$A : B :: C : D$ which gives $A \times D = B \times C$
$A : B :: E : F$ which gives $A \times F = B \times E$
$A : B :: G : H$ which gives $A \times H = B \times G$

We shall then have, by addition,

$$A \times (D+F+H) = B \times (C+E+G);$$

and consequently, by Th. II.

$$A : B :: C+E+G : D+F+H.$$

Sch. The proposition may be verified by the following proportions: viz.

$2 : 4 :: 6 : 12$ and $1 : 2 :: 3 : 6$

Then, $2 : 4 :: 6+3 : 12+6$;

that is, $2 : 4 :: 9 : 18$,

in which the ratio is still 2.

THEOREM XIII.

If four quantities are in proportion, their squares or cubes will also be proportional.

If we have the proportion

$$A : B :: C : D,$$

it gives

$$\frac{B}{A}=\frac{D}{C}$$

Then, if we square both members, we have

$$\frac{B^2}{A^2}=\frac{D^2}{C^2}$$

and if we cube both members, we have

$$\frac{B^3}{A^3}=\frac{D^3}{C^3}$$

and then, changing these equalities into a proportion, we have for the first,

$$A^2 : B^2 :: C^2 : D^2;$$

and for the second

$$A^3 : B^3 :: C^3 : D^3.$$

Sch. We may verify the proposition by the proportion,

$$2 : 4 :: 6 : 12,$$

and by squaring each term we have,

$$4 : 16 :: 36 : 144$$

numbers which are still proportional, and in which the ratio is 4.

If we cube the numbers we have,

$$2^3 \; : \; 4^3 \; :: \; 6^3 \; : \; 12^3$$

that is, $8 \; : \; 64 \; :: \; 216 \; : \; 1728,$

in which the ratio is 8.

THEOREM XIV.

If we have two sets of proportional quantities, the products of the corresponding terms will be proportional.

Let us take the proportions,

$A \; : \; B \; :: \; C \; : \; D$ which gives $\frac{B}{A}=\frac{D}{C}$

$E \; : \; F \; :: \; G \; : \; H$ which gives $\frac{F}{E}=\frac{H}{G}$

Multiplying the equalities together, we have

$$\frac{B\times F}{A\times E}=\frac{D\times H}{C\times G}$$

and this by Th. II, gives

$$A\times E \; : \; B\times F \; :: \; C\times G \; : \; D\times H.$$

Sch. The proposition may be verified by the following proportions:

$$8 \; : \; 12 \; :: \; 10 \; : \; 15,$$

and $3 \; : \; 4 \; :: \; 6 \; : \; 8;$

we shall then have

$$24 \; : \; 48 \; :: \; 60 \; : \; 120$$

which are proportional, the ratio being 2.

GEOMETRY.

BOOK IV

OF THE MEASUREMENT OF AREAS, AND THE PROPORTIONS OF FIGURES.

DEFINITIONS.

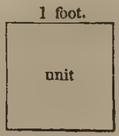
1 Similar figures, are those which have the angles of the one equal to the angles of the other, each to each, and the sides about the equal angles proportional.

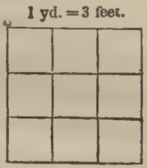
2. Any two sides, or any two angles, which are like placed in the two similar figures, are called *homologous* sides or angles.

3. A polygon which has all its angles equal, each to each, and all its sides equal, each to each, is called a *regular polygon.* A regular polygon is both equiangular and equilateral.

4. If the length of a line be computed in feet; one foot is the unit of the line, and is called the *linear unit.* If the length of a line be computed in yards, one yard is the linear unit

5. If we describe a square on the unit of length, such square is called the unit of surface. Thus, if the linear unit is one foot, one square foot will be the unit of surface, or superficial unit.





6. If the linear unit is one yard, one square yard will be the unit of surface; and this square yard contains nine square feet.

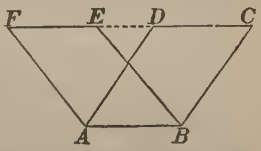
7. The *area* of a figure is the measure of its surface. The unit of the number which expresses the area, is a square, the side of which is the unit of length.

8. Figures have equal areas, when they contain the same measuring unit an equal number of times.

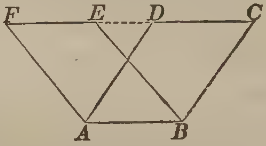
9. Figures which have equal areas are called *equivalent.* The term *equal*, when applied to figures, implies an equality in all respects. The term *equivalent*, implies an equality in one respect only: viz. an equality in their areas. The sign ⇔, denotes equivalency, and is read, *is equivalent to.*

THEOREM I.

Parallelograms which have equal bases and equal altitudes, are equivalent.



Place the base of one parallelogram on that of the other, so that *AB* shall be the common base of the two parallelograms *ABCD* and *ABEF.* Now, since the parallelograms have the same altitude, their upper bases, *DC* and *FE*, will fall on the same line *FEDC*, parallel to *AB.* Since the opposite sides of a parallelogram are equal to each other (Bk. I Th. xxiii), *AD* is equal to *BC.* Also, *DC* and *FE* are each equal to *AB:* and consequently, they are equal to each

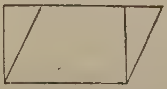


other (Ax. 1). To each, add ED: then will CE be equal to DF.

But since the line FC cuts the two parallels CB and DA, the angle BCE will be equal to the angle ADF (Bk. I. Th. xiv): hence, the two triangles ADF and BCE have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each; consequently, they are equal (Bk. I. Th. iv).

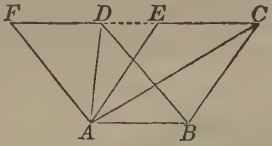
If then, from the whole space $ABCF$ we take away the triangle ADF, there will remain the parallellogram $ABCD$; but if we take away the equal triangle BEC, there will remain the parallelogram $ABEF$: hence, the parallelogram $ABEF$ is equivalent to the parallelogram $ABCD$ (Ax. 3).

Cor. A parallelogram and a rectangle, having equal bases and equal altitudes, are equivalent.



THEOREM II.

Triangles which have equal bases and equal altitudes, are equivalent.



Place the base of one triangle on that of the other, so that ABC and ABD shall be two triangles, having a common base AB, and for their altitude, the distance between the two parallels AB, FC: then will the triangle ABC be equivalent to the triangle ADB.

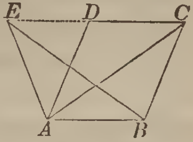
For, through A draw AE parallel to BC, and AF parallel to BD, forming the two parallelograms BE and BF. Then, since these parallelograms have a common base and equal altitudes, they will be equivalent (Th. i).

But the triangle *ABC* is half the parallelogram *BE* (Bk. I. Th. xxiii); and *ABD* is half the equal parallelogram *BF*. hence, the triangle *ABC* is equivalent to the triangle *ABD*.

THEOREM III.

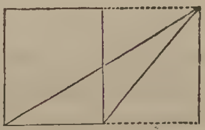
If a triangle and a parallelogram have equal bases and equal altitudes, the triangle will be half the parallelogram.

Place the base of the triangle on the base of the parallelogram, so that *AB* shall be the common base of the triangle and parallelogram: then will the triangle *ABE* be half the parallelogram *BD*.



For, draw the diagonal *AC*. Then, since the altitude of the triangle *AEB* is equal to that of the parallelogram, the vertex will be found some where in *CD*, or in *CD* produced. Now the two triangles *ABC* and *ABE*, having the same base *AB*, and equal altitudes, are equivalent (Th. ii). But the triangle *ABC* is half the parallelogram *BD* (Bk. I. Th. xxiii): hence, the triangle *ABE* is half the parallelogram *BD* (Ax. 1).

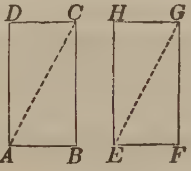
Cor. Hence, if a triangle and a rectangle have equal bases and equal altitudes, the triangle will be half the rectangle.



For the rectangle would be equivalent to a parallelogram of the same base and altitude (Th. i. Cor.), and since the triangle is half the parallelogram, it is also equivalent to half the rectangle

THEOREM IV.

Rectangles which are described on equal lines are equivalent.



Let *BD* and *FH* be two rectangles, having the sides *AB*, *BC*, equal to the two sides *EF*, *FG*, each to each: then will the rectangle *ABCD*, described on the lines *AB*, *BC*, be equivalent to the rectangle *EFGH*, described on the lines *EF*, *FG*.

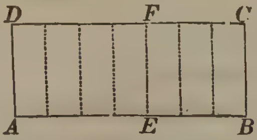
For, draw the diagonals *AC*, *EG*, dividing each parallelogram into two equal parts.

Then the two triangles, *ABC*, *EFG*, having two sides and the included angle of the one equal to two sides and the included angle of the other, each to each, are equal (Bk. I. Th. iv). But these equal triangles are halves of the respective rectangles (Th. iii. Cor.): hence, the rectangles are equal (Ax. 7); and consequently equivalent.

Cor. The squares on equal lines are equal. For a square is but a rectangle having its sides equal.

THEOREM V.

Two rectangles having equal altitudes are to each other as their bases.



Let *AEFD* and *EBCF* be two rectangles having the common altitude *AD*; then will they be to each other as the bases *AE* and *EB*.

For, suppose the base *AE* to be to the base *EB*, as any two numbers, say the numbers 4 and 3. Let *AE* be then divided into four equal parts, and *EB* into three equal parts, and through the points of division draw parallels to *AD*. We shall thus form seven rectangles, all equivalent to each other since they have equal bases and equal altitudes (Th. iv).

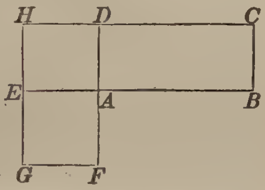
But the rectangle *AEFD* will contain four of these partial rectangles, while the rectangle *EBCF* will contain three; hence, the rectangle *AEFD* will be to the rectangle *EBCF* as 4 to 3; that is, as the base *AE* to the base *EB*.

The same reasoning may be applied to any other rectangles whose bases are whole numbers: hence,

$$AEFD : EBCF :: AE : EB.$$

THEOREM VI.

Any two rectangles are to each other as the products of their bases and altitudes.

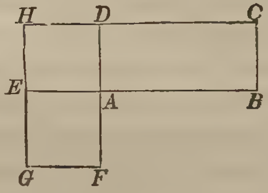


Let *ABCD* and *AEGF* be two rectangles: then will $ABCD : AEGF :: AB \times AD : AF \times AE$

For, having placed the two rectangles so that *BAE* and *DAF* shall form straight lines, produce the sides *CD* and *GE* until they meet in *H*.

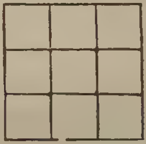
Then, the two rectangles *ABCD*, *AEHD*, having the common altitude *AD*, are to each other as their bases *AB* and *AE* (Th. v). In like manner, the two rectangles *AEHD*, *AEGF*, having the same altitude *AE*, are to each other as their bases *AD* and *AF*. Thus, we have the proportions

$$ABCD : AEHD :: AB : AE,$$
$$AEHD : AEGF :: AD : AF.$$



If, now, we multiply the corresponding terms together, the products will be proportional (Bk. III. Th. xiv.); and the common multiplier *AEHD* may be omitted (Bk. III. Th. ix.): hence, we shall have

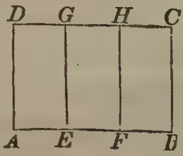
$$ABCD : AEGF :: AB \times AD : AE \times AF.$$



Sch. Hence, the product of the base by the altitude may be assumed as the measure of a rectangle. This product will give the number of superficial units in the surface: because, for one unit in height, there are as many superficial units as there are linear units in the base; for two units in height, twice as many; for three units in height, three times as many, &c.

THEOREM VII.

The sum of the rectangles contained by one line, and the several parts of another line any way divided, is equivalent to the rectangle contained by the two whole lines.



Let *AD* be one line, and *AB* the other, divided into the parts *AE*, *EF*, *FB*: then will the rectangles contained by *AD* and *AE*, *AD* and *EF*, *AD* and *FB*, be equivalent to the rectangle *AC* which is contained by the lines *AD* and *AB*.

For, through *E* and *F* draw

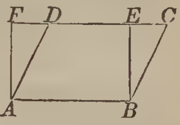
be equal to the rectangle of $AD \times AE$; EH will be equal to $EG \times EF$, or to $AD \times EF$; and FC will be equal to $FH \times FB$, or to $AD \times FB$.

But the rectangle AC is equal to the sum of the partial rectangles: hence,

$$AD \times AB \mathrel{=\!\!\!\bigcirc\!\!\!=} AD \times AE + AD \times EF + AD \times FB.$$

THEOREM VIII.

The area of any parallelogram is equal to the product of its base by its altitude.



Let $ABCD$ be any parallelogram, and BE its altitude: then will its area be equal to $AB \times BE$.

For, draw AF perpendicular to the base AB, and produce CD to F. Then, the parallelogram BD and the rectangle BF, having the same base and altitude are equivalent (Th. i. Cor.). But the area of the rectangle BF is equal to the product of its base AB by the altitude AF (Th. vi. Sch.): hence, the area of the parallelogram is equal to $AB \times BE$.

Cor. Parallelograms of equal bases are to each other as their altitudes; and if their altitudes are equal, they are to each other as their bases.

For, let B be the common base, and C and D the altitudes of two parallelograms. Then, by the theorem, their areas are to each other, as

$$B \times C \ : \ B \times D,$$

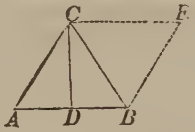
that is, (Bk. III. Th ix), as $C : D$

If A and B be their bases, and C their common altitude, then they will be to each other as

$A \times C \ : \ B \times C$: that is, as $A \ : \ B$.

THEOREM IX.

The area of a triangle is equal to half the product of its base by its altitude.



Let ABC be any triangle and CD its altitude: then will its area be equal to half the product of $AB \times CD$.

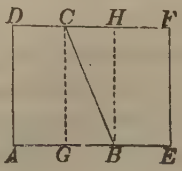
For, through B draw BE parallel to AC, and through C draw CE parallel to AB: we shall then form the parallelogram AE, having the same base and altitude as the triangle ABC.

But the area of the parallelogram is equal to the product of the base AB by its altitude DC; and since the parallelogram is double the triangle (Th. iii), it follows that the area of the triangle is equal to half this product: that is, to half the product of $AB \times CD$.

Cor. Two triangles of the same altitude are to each other as their bases; and two triangles of the same base are to each other as their altitudes. And generally, triangles are to each other as the products of their bases and altitudes.

THEOREM X.

The area of a trapezoid is equal to half the product of its altitude multiplied by the sum of its parallel sides.



Let $ABCD$ be a trapezoid, CG its altitude, and AB, DC its parallel sides: then will its area be equal to half the product of

$CG \times (AB+DC)$.

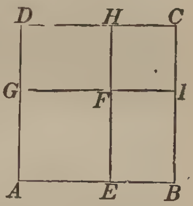
For, produce AB until BE is equal to DC, and complete the rectangle AF; also, draw BH perpendicular to AB.

Then, the rectangle AC will be equivalent to BF, since they have equal bases and equal altitudes (Th iv). The diagonal BC will divide the rectangle GH into two equal triangles; and hence, the trapezoid $ABCD$ will be equivalent to the trapezoid $BEFC$; and consequently, the rectangle AF, is double the trapezoid $ABCD$.

But the rectangle AF is equivalent to the product of $AD \times AE$; that is, to $CG \times (AB + DC)$; and consequently, the trapezoid $ABCD$ is equal to half that product.

THEOREM XI.

If a line be divided into two parts, the square described on the whole line is equivalent to the sum of the squares described on the two parts, together with twice the rectangle contained by the parts.

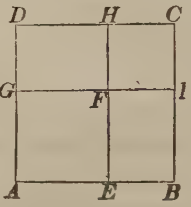


Let the line AB be divided into two parts at the point E: then will the square described on AB be equivalent to the two squares described on AE and EB, together with twice the rectangle contained by AE and EB: that is

$$\overline{AB}^2 \approx \overline{AE}^2 + \overline{EB}^2 + 2AE \times EB.$$

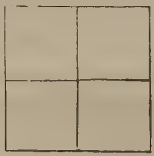
For let AC be a square on AB, and AF a square on AE. and produce the sides EF and GF to H and I.

Then, since EH is equal to AD, being the opposite side of a rectangle, it is also equal to AB; and GI is likewise equal to AB If, therefore, from these equals we take away EF and



GF, there will remain *FH* equal to *FI*, and each will be equal to *HC* or *IC*; and since the angle at *F* is a right angle, it follows that *FC* is equal to a square described on *EB*. It also follows, that *DF* and *FB* are each equal to the rectangle of *AE* into *EB*.

But the square *ABCD* is made up of four parts, viz., the square on *AE*; the square on *EB*; the rectangle *DF*, and the rectangle *FB*. Hence, the square on *AB* is equivalent to the square on *AE* plus the square on *EB*, plus twice the rectangle contained by *AE* and *EB*.



Cor. If the line *AB* be divided into two equal parts, the rectangles *DF* and *FB* would become squares, and the square described on the whole line would be equivalent to four times the square described on half the line.

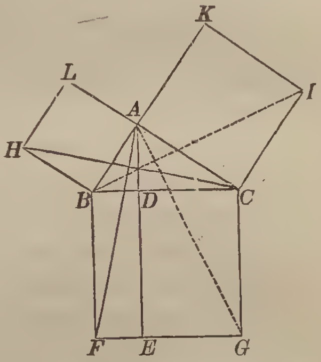
Sch. The property may be expressed in the language of algebra, thus,

$$(a+b)^2=a^2+2ab+b^2.$$

THEOREM XII.

The square described on the hypothenuse of a right angled triangle, is equivalent to the sum of the squares described on the other two sides.

Of Right Angled Triangles.



Let *BAC* be a right angled triangle, right angled at *A*: then will the square described on the hypothenuse *BC*, be equivalent to the two squares described on *BA* and *AC*.

Having described the squares *BG*, *BL*, and *AI*, let fall from *A*, on the hypothenuse, the perpendicular *AD*, and produce it to *E*; then draw the diagonals *AF*, *CH*.

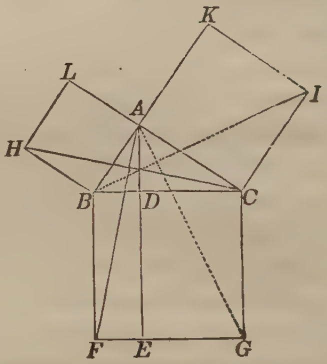
Now, the angle *ABF* is made up of the right angle *FBC* and the angle *CBA*; and the angle *CBH* is made up of the right angle *ABH* and the same angle *CBA*: hence, the angle *ABF* is equal to *CBH*. But *FB* is equal to *BC*, being sides of the same square; and for a like reason, *BA* is equal to *HB*. Therefore, the two triangles *ABF* and *CBH*, having two sides and the included angle of the one equal to two sides and the included angle of the other, each to each, are equal (Bk. I. Th. iv).

Since the angles *BAC* and *BAL* are right angles, as also the angle *ABH*, it follows that *CAL* is a straight line parallel to *BH*. (Bk. I. Th. ii. Cor. 3). Hence, the square *HA* and the triangle *HBC* stand on the same base and between the same parallels; therefore the triangle is half the square (Th. iii. Cor.). For a like reason, the triangle *ABF* is half the rectangle *BE*.

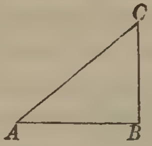
But it has already been proved that the triangle *ABF* is equal to the triangle *CBH*: hence, the rectangle *BE*, which is double the former, is equivalent to the square *BL*, which is double the latter (Ax. 6).

In the same manner it may be proved, that the rectangle *DG* is equivalent to the square *CK*.

But the two rectangles *BE*, *DG*, make up the square *BG*: therefore, the square *BG*, described on the hypothenuse, is equivalent to the squares *BL* and *CK*, described on the other two sides.



Cor. Hence, the square of either side of a right angled triangle is equivalent to the square of the hypothenuse diminished by the square of the other side. That is, in the right angled triangle *ABC*

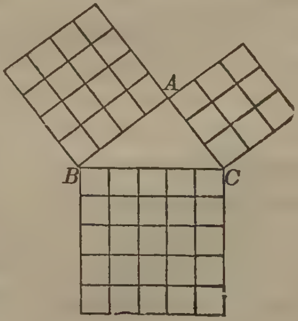


$$\overline{AB}^2 \asymp \overline{AC}^2 - \overline{BC}^2$$

or

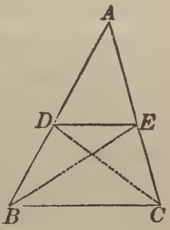
$$\overline{BC}^2 \asymp \overline{AC}^2 - \overline{AB}^2$$

Sch. The last theorem may be illustrated by describing a square on the hypothenuse *BC*, equal to 5, also on the sides *BA*, *AC*, respectively equal to 4 and 3; and observing that the number of small squares in the large square is equal to the number in the two small squares.



THEOREM XIII.

If a line be drawn parallel to the base of a triangle, it will divide the other two sides proportionally.



Let ABC be any triangle, and DE a straight line drawn parallel to the base BC: then will

$$AD : DB :: AE : EC.$$

For, draw BE and DC. Then, the two triangles BDE and DCE have the same base DE, and the same altitude, since their vertices B and C, lie in the line BC parallel to DE: hence, they are equivalent (Th. ii).

Again, the triangles ADE and BDE, have a common vertex E, and the same altitude; consequently, they are to each other as their bases (Th. ix. Cor.); hence, we have

$$ADE : BDE :: AD : DB.$$

But the triangles ADE and CDE, having a common vertex D, are to each other as their bases AE and EC: hence, we have

$$ADE : CDE :: AE : EC.$$

But the triangles BDE and CDE have been proved equivalent: hence, in the two proportions, the first antecedent and consequent in each are equal: therefore, by (Bk. III. Th. v) we have

$$AD : BD :: AE : EC.$$

Cor. The sides AB, AC, are also proportional to the parts AD, AE, or to BD, CE.

For, by composition (Bk. III. Th. vii), we have

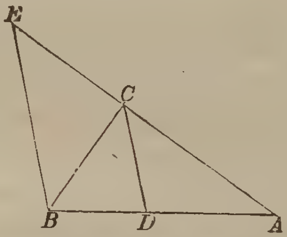
$$AD+BD : BD :: AE+EC : EC.$$

Then, by alternation (Bk. III. Th. iv).

$AB : AC :: BD : EC$, hence, also, $AB : AC :: AD : AE$

THEOREM XIV.

A line which bisects the vertical angle of a triangle divides the base into two segments which are proportional to the adjacent side.



Let ACB be a triangle, having the angle C bisected by the line CD: then will

$$AD : DR :: AC : CB.$$

For, draw BE parallel to CD and produce AC to E. Then, since CB cuts the two parallels CD, EB, the alternate angles BCD and CBE are equal (Bk. I. Th. xii): hence, CBE is equal to angle ACD.

But, since AE cuts the two parallels CD, BE, the angle ACD is equal to CEB (Bk. I. Th. xiv): consequently, the angle CBE is equal to the angle CEB (Ax. I): hence, the side CB is equal to CE (Bk. I. Th. vii.)

Now, in the triangle ABE the line CD is drawn parallel to BE: hence, by the last theorem, we have

$$AD : DB :: AC : CE,$$

and by placing for CE, its equal CB, we have

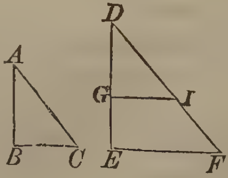
$$AD : DB :: AC : CB.$$

THEOREM XV.

Equiangular triangles have their sides proportional, and are similar.

Let ABC and DEF be two equiangular triangles, having the angle A equal to the angle D, the angle C to the angle F, and the angle B to the angle E: then will

$$AB : AC :: DE : DF$$



For, on the sides of the larger triangle DEF, make DI equal to AC and DG equal to AB, and join IG. Then the two triangles ABC and DIG, having two sides and the included angle of the one equal to two sides and the included angle of the other, each to each, will be equal (Bk. I Th. iv) Hence, the angles I and G are equal to C and B, and consequently, to the angles F and E: therefore, IG is parallel to EF (Bk. I. Th. xiv, Cor. 1).

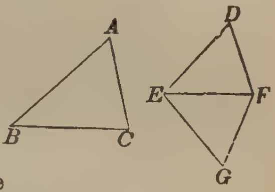
Now, in the triangle DEF, since IG is parallel to the base, we have (Th. xiii).

$$DG : DI :: DE : DF,$$

that is, $$AB : AC :: DE : DF.$$

THEOREM XVI.

Two triangles which have their sides proportional are equiangular and similar.



Let BAC and EDF be two triangles having

$$BC : EF :: AB : ED,$$

and $$BC : EF :: AC : DF;$$

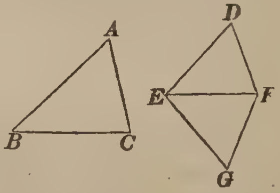
then will they have the corresponding angles equal, viz., the angle

$$B=E, \quad A=D \quad \text{and} \quad C=F.$$

For, at the point E make FEG equal to the angle B, and at F make the angle EFG equal to the angle C: Then will the angle at G be equal to A, and the two triangles BAC and EGF will be equiangular (Bk. I. Th. xvii. Cor 1).

Therefore, by the last theorem we shall have

$$BC : EF :: AB : EG;$$



but by hypothesis,

$$BC : EF :: AB : DE:$$

hence, EG is equal to ED.

By the last theorem we also have

$$BC : EF :: AC : FG.$$

and by hypothesis,

$$BC : EF :: AC : DF;$$

hence, FG is equal to DF.

Therefore, the triangles DEF and EGF, having their three sides equal, each to each, are equiangular (Bk. I. Th. viii). But, by construction, the triangle EFG is equiangular with BAC: hence, the triangles BAC and EDF are equiangular, and consequently they are similar.

Sch. By Theorem XV, it appears that if the corresponding angles of two triangles are equal, each to each, the corresponding sides will be proportional; and in the last theorem it was proved that if the sides are proportional, the corresponding angles will be equal.

Now, these proportions do not hold good in the quadrilaterals. For, in the square and rectangle, the corresponding angles are equal, but the sides are not proportional; and the angles of a parallelogram or quadrilateral, may be varied at pleasure, without altering the lengths of the sides.

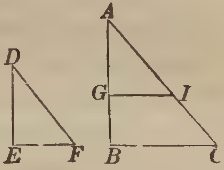
THEOREM XVII.

If two triangles have an angle in the one equal to an angle in the other, and the sides containing these angles proportional, the two triangles will be equiangular and similar.

Let ABC and DEF be two triangles having the angle A equal to the angle D, and

$$AB : DE :: AC : DF;$$

then will the two triangles be similar.



For, lay off AG equal to DE, and through G draw GI parallel to BC. Then the angle AGI will be equal to the angle ABC (Bk. I. Th. xiv); and the triangles AGI and ABC will be equiangular. Hence, we shall have

$$AB : AG :: AC : AI.$$

But, by hypothesis, we have

$$AB : DE :: AC : DF,$$

and by construction, AG is equal to DE; therefore, AI is equal to DF, and consequently, the two triangles AGI and DEF are equal in all their parts (Bk. I. Th. iv). But the triangle ABC is similar to AGI, consequently it is similar to DEF

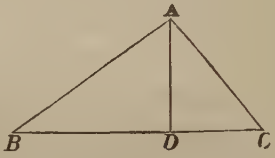
THEOREM XVIII.

If from the right angle of a right angled triangle, a perpendicular be let fall on the hypothenuse, then

I. *The two partial triangles thus formed will be similar to each other and to the whole triangle.*

II. *Either side including the right angle will be a mean proportional between the hypothenuse and the adjacent segment.*

III. *The perpendicular will be a mean proportional between the segments of the hypothenuse*



Let ABC be a right angled triangle, and AD perpendicular to the hypothenuse.

The two triangles BAC and BAD having the common angle B, and the right angle BAC equal to the right angle at D, will be equiangular (Bk. I. Th. xvii Cor. 1); and, consequently, similar (Th. xv). For a like reason the triangles BAC and CAD are similar.

Now, from the triangles BAC and BAD, we have

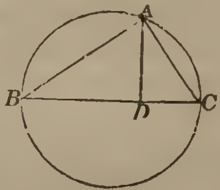
$$BC : BA :: BA : BD.$$

From the triangles BAC and CAD, we have

$$BC : CA :: CA : CD:$$

and from the triangles BAD and DAC, we have

$$BD : AD :: AD : DC.$$



Cor. If from a point A, in the circumference of a circle, AD be drawn perpendicular to any diameter as BC, and the chords AB AC be also drawn, then the angle BAC will be a right angle (Bk. II. Th. x): and by the theorem we shall have,

1st The perpendicular AD a mean proportional between the segments BD and DC.

2d Each chord will be a mean proportional between the diameter and the adjacent segment.

That is,

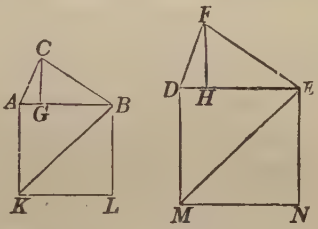
$$\overline{AD}^2 = BD \times DC$$
$$\overline{AB}^2 = BC \times BD$$
$$\overline{AC}^2 = BC \times CD$$

THEOREM XIX.

Similar triangles are to each other as the squares described on their homologous sides

Let ABC and DEF be two similar triangles, and AL and DN the squares described on the homologous sides AB, DE: then will the triangle

$$ABC : DEF :: AL : DN.$$



For, draw CG and FH perpendicular to the bases AB, DE, and draw the diagonals BK and EM.

Then, the similar triangles ABC and DEF, having their homologous sides proportional, we have

$$AC : DF :: AB : DE;$$

and the two ACG, DFH, give

$$AC : DF :: CG : FH;$$

hence, (Bk. III. Th. v), we have

$$AB : DE :: CG : FH,$$

or (Bk. III. Th. iv),

$$AB : CG :: DE : FH.$$

Now, the two triangles ABC and AKB have the common base AB; and the triangles DEF and DEM have the common base DE; and since triangles on equal bases are to each other as their altitudes (Th. ix, Cor.), we have

the triangle

$$ABC : ABK :: CG : AK \text{ or } AB$$

and the triangle,

$$DEF : DME :: FH : DM \text{ or } DE.$$

But we have proved

$$CG : AB :: FH : DE;$$

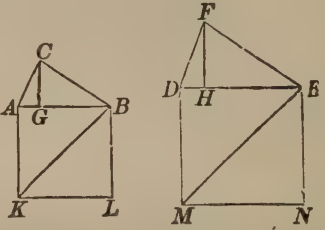
hence, $ABC : ABK :: DEF : DME$,

or alternately,

$$ABC : DEF :: ABK : DME.$$

But the squares AL and DN being each double of the triangles AKB and DME have the same ratio; hence,

$$ABC : DEF :: AL : DN.$$

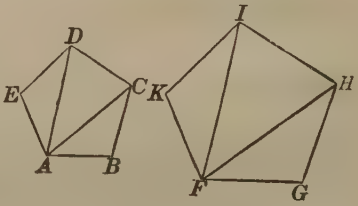


THEOREM XX.

Two similar polygons may be divided into an equal number of triangles, similar each to each, and similarly placed.

Let $ABCDE$ and $FGHIK$ be two similar polygons.

From the angle A draw the diagonals AC, AD: and from the homologous angle F, draw FH, FI.



Now, since the polygons are similar, the homologous angles B and G will be equal, and the sides about the equal angles proportional (Def. 1): that is,

$$AB : BC :: FG : GH.$$

Hence, the triangles ABC and FGH have an angle in each equal, and the sides about the equal angles proportional: therefore, they are similar (Th. xvii), and consequently, the angle ACB is equal to FHG. Taking these from the equal angles BCD and GHI, there will remain ACD equal to FHI. The two triangles ACD and FHI will then have an angle in each equal, and the sides about the equal angles proportional: hence, they will be similar.

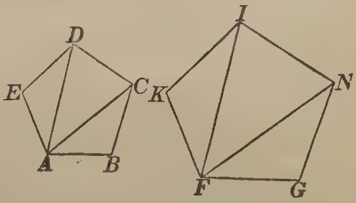
In the same manner it may be shown that the triangles AED and FKI are similar: and, hence, whatever be the number of sides of the polygons, they may be divided into an equal number of similar triangles.

THEOREM XXI.

Similar polygons are to each other as the squares described on their homologous sides.

Let $ABCDE$ and $FGNIK$, be two similar polygons; then will they be to each other as the squares described on AB, FG, or any other two homologous sides.

For, let the polygons be divided, as in the last theorem, into an equal number of similar triangles. Then, by Theorem XIX, we have the triangles



$$ABC : FGN :: \overline{AB}^2 : \overline{FG}^2$$
$$ADC : FIN :: \overline{DC}^2 : \overline{IN}^2$$
$$ADE : FIK :: \overline{DE}^2 : \overline{IK}^2$$

But since the polygons are similar, the ratio of the last antecedent to its consequent, in each of the proportions, is the same: hence, we have (Bk. III. Th. xii).

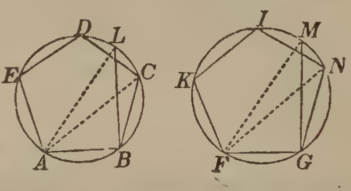
$$ABC+ADC+ADE : FGN+FIN+FIK :: \overline{AB}^2 : \overline{FG}^2;$$

that is, $ABCDE : FGNIK :: \overline{AB}^2 : \overline{FG}^2;$

Hence, the areas of similar polygons are to each other as the squares described on their homologous sides

THEOREM XXII.

If similar polygons are inscribed in circles, their homologous sides, and also their perimeters, will have the same ratio to each other as the diameters of the circles in which they are inscribed



Let $ABCDE$, $FGNIK$, be two similar figures, inscribed in the circles whose diameters are AL and FM: then will each side, AB, BC, &c., of the one, be to the homologous side FG, GN, &c., of the other, as the diameter AL to the diameter FM. Also, the perimeter $AB+BC+CD$ &c., will be to the perimeter $FG+GN+NI$ &c., as the diameter AL to the diameter FM

For, draw the two corresponding diagonals AC, FN, as also the lines BL and GM.

Then, the two triangles ACB and FNG will be similar (Th. xx); and therefore, the angle ACB is equal to the angle FNG. But, the angle ACB is equal to the angle ALB, and the angle FNG to the angle FMG (Bk. II. Th. ix): hence, the angle ALB is equal to the angle FMG (Ax. 1); and since ABL and FGM are right angles (Bk. II. Th. x), the two triangles ALB and FMG will be equiangular (Bk. I. Th. **xvii.** Cor. 1), and consequently similar (Th. xv).

Therefore,

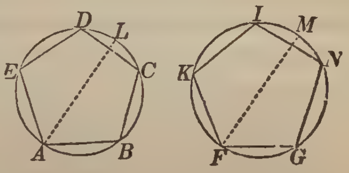
$$AB : FG :: AL : FM.$$

Again, since any two homologous sides are to each other in the same ratio as AL to FM, we have (Bk. III. Th xii),

$$AB+BC+CD \text{ \&c.} : FG+GN+NI \text{ \&c.} :: AL : FM.$$

THEOREM XXIII.

Similar polygons inscribed in circles are to each other as the squares of the diameters of the circles.



Let *ABCDE*, *FGNIK*, be two polygons inscribed in the circles whose diameters are *AL* and *FM*: then will the polygon *ABCDE*, be to the polygon *FGNIK* as the square of *AL* to the square of *FM*.

For, the polygons being similar, are to each other as the squares of their like sides (Th. xxi); that is, as $\overline{AB}^2$ to $\overline{FG}^2$.

But, by the last theorem,

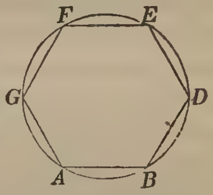
$$AB \ : \ FG \ :: \ AL \ : \ FM;$$

therefore (Bk III. Th. xiii),

$$\overline{AB}^2 \ : \ \overline{FG}^2 \ :: \ \overline{AL}^2 \ : \ \overline{FM}^2;$$

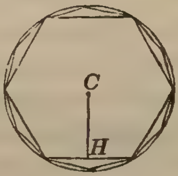
consequently,

$$ABCDE \ : \ FGNIK \ :: \ \overline{AL}^2 \ : \ \overline{FM}^2.$$



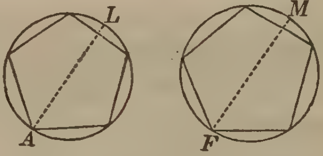
Sch. If any regular polygon, *ABDEFG*, be inscribed in a circle, and then the arcs *AB*, *BE*, &c., be bisected, and lines be drawn through these points of bisection, a new polygon will be formed having double the number of sides. It is plain that this new polygon will differ less from the circle than the first polygon, and its sides will lie nearer the circumference than the sides of the first polygon.

If now, we suppose the number of sides to be continually increased, the length of each side will constantly diminish until finally the polygon will become equal to the circle, and the perimeter will coincide with the circumference. When this takes place, the line *CH* drawn perpendicular to one of the sides, will become equal to the radius of the circle.



THEOREM XXIV.

The circumferences of circles are to each other as their diameters



Let there be two circles whose diameters are *AL* and *FM:* then will their circumferences be to each other as *AL* to *FM.*

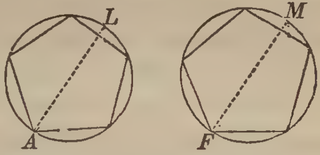
For, suppose two similar polygons to be inscribed in the circles: their perimeters will be to each other as *AL* to *FM* (Th. xxii).

Let us now suppose the arcs which subtend the sides of the polygons to be bisected, and new polygons of double the number of sides to be formed: their perimeters will still be to each other as *AL* to *FM*, and if the number of sides be increased until the perimeters coincide with the circumference, we shall have the circumferences to each other as the diameters *AL* and *FM.*

THEOREM XXV.

The areas of circles are to each other as the squares of their diameters.

Let there be two circles whose diameters are *AL* and *FM:* then will their areas be to each other as the square of *AL* to the square of *FM*.



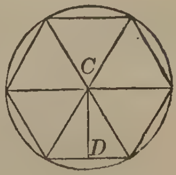
For, suppose two similar polygons to be inscribed in the circles: then will they be to each other as $\overline{AL}^2$ to $\overline{FM}^2$ (Th xxiii).

Let us now suppose the number of sides of the polygons to be increased, by bisecting the arcs, until their perimeters shall coincide with the circumferences of the circles. The polygons will then become equal to the circles,and hence, the areas of the circles will be to each other as the squares of their diameters.

Cor. Since the circumferences of circles are to each other as their diameters (Th. xxiv), it follows, that the areas which are proportional to the squares of the diameters, will also be proportional to the squares of the circumferences

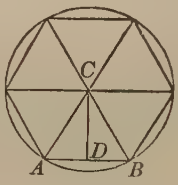
THEOREM XXVI.

The area of a regular polygon inscribed in a circle, is equal to half the product of the perimeter and the perpendicular let fall from the centre on one of the sides.



Let *C* be the centre of a circle circumscribing the regular polygon, and *CD* a perpendicular to one of its sides: then will its area be equal to half the product of *CD* by the perimeter.

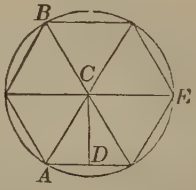
For, from *C* draw radii to the vertices of the angles, forming as many equal triangles as the polygon has sides, in each of which the perpendicular on the base will be equal to *CD*. Now, the area of one of them, as *ACB*, will be equal to half the product of *CD* by the base *AB*; and the same will be true for each of the other triangles: hence, the area of the polygon will be equal to half the product of *CD* by the perimeter



THEOREM XXVII.

The area of a circle is equal to half the product of the radius by the circumference.

Let *C* be the centre of a circle: then will its area be equal to half the product of the radius *AC* by the circumference *ABE*.



For, inscribe within the circle a regular hexagon, and draw *CD* perpendicular to one of its sides. Then, the area of the polygon will be equal to half the product of *CD* multiplied by the perimeter (Th. xxvi).

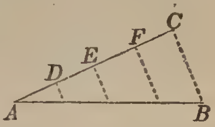
Let us now suppose the number of sides of the polygon to be increased, until the perimeter shall coincide with the circumference; the polygon will then become equal to the circle, and the perpendicular *CD* to the radius *CA*. Hence, the area of the circle will be equal to half the product of the radius by the circumference.

PROBLEMS

RELATING TO THE FOURTH BOOK.

PROBLEM I.

To divide a line into any proposed number of equal parts

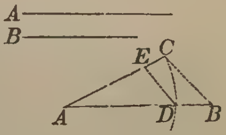


Let AB be the line, and let it be required to divide it into four equal parts.

Draw any other line, AC, forming an angle with AB, and take any distance, as AD, and lay it off four times on AC. Join C and B, and through the points D, E, and F, draw parallels to CB These parallels to BC will divide the line AB into parts proportional to the divisions on AC (Th. xiii): that is, into equal parts.

PROBLEM II.

To find a third proportional to two given lines.



Let A and B be the given lines.

Make AB equal to A, and draw AC, making an angle with it. On AC lay off AC equal to B, and join BC. then lay off AD, also equal to B, and through D draw DE parallel to BC: then will AE be the third proportional sought.

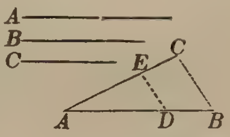
For, since DE is parallel to BC, we have (Th. xiii)

$$AB : AC :: AD \text{ or } AC : AE;$$

therefore, AE is the third proportional sought.

PROBLEM III.

To find a fourth proportional to the lines A, B, and C.



Place two of the lines forming an angle with each other at A; that is, make AB equal to A, and AC equal B; also, lay off AD equal to C. Then join BC, and through D draw DE parallel to BC, and AE will be the fourth proportional sought.

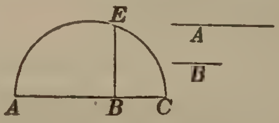
For, since DE is parallel to BC, we have

$$AB \ : \ AC \ :: \ AD \ : \ AE;$$

therefore, AE is the fourth proportional sought.

PROBLEM IV.

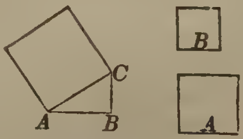
To find a mean proportional between two given lines, A and B.



Make AB equal to A, and BC equal to B: on AC describe a semicircle. Through B draw BE perpendicular to AC, and it will be the mean proportional sought (Th. xviii. Cor).

PROBLEM V.

To make a square which shall be equivalent to the sum of two given squares.

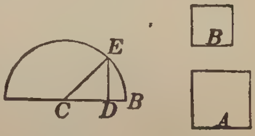


Let A and B be the sides of the given squares.

Draw an indefinite line AB, and make AB equal to A. At B draw BC perpendicular to AB, and make BC equal to B: then draw AC and the square described on AC will be equivalent to the squares on A and B (Th. xii).

PROBLEM VI.

To make a square which shall be equivalent to the difference between two given squares.



Let A and B be the sides of the given squares.

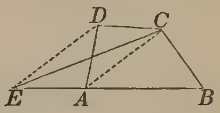
Draw an indefinite line, and make CB equal to A, and CD equal to B. At D draw DE perpendicular to CB, and with C as a centre, and CB as a radius, describe a semicircle meeting DE in E, and join CE: then will the square described on ED be equal to the difference between the given squares.

For, CE is equal to CB, that is, equal to A, and CD is equal to B: and by (Th. xii. Cor.),

$$\overline{ED}^2=\overline{CE}^2-\overline{CD}^2.$$

PROBLEM VII.

To make a triangle which shall be equivalent to a given quadrilateral.



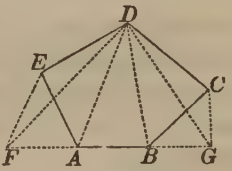
Let $ABCD$ be the given quadrilateral.

Draw the diagonal AC, and through D draw DE parallel to AC, meeting BA produced at E. Join EC: then will the triangle CEB be equivalent to the quadrilateral BD.

For, the two triangles ACE and ADC, having the same base AC, and the vertices of the angles D and E in the same line DE parallel to AC, are equivalent (Th. ii). If to each, we add ACB, we shall then have the triangle ECB equivalent to the quadrilateral BD (Ax. 2).

PROBLEM VIII.

To make a triangle which shall be equivalent to a given polygon.



Let $ABCDE$ be the polygon.

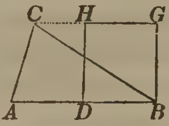
Draw the diagonals AD, BD. Produce AB in both directions, and through C and E draw CG and EF, respectively parallel to AD and BD: then join FD and DG, and the triangle FDG will be equivalent to the polygon $ABCDE$.

For, the triangle AED is equivalent to the triangle AFD and DBC to DBG (Th. ii); and by adding ADB to the equals, we shall have the triangle FDG equivalent to the polygon $ABCDE$.

PROBLEM IX.

To make a rectangle that shall be equivalent to a given triangle.

Let ABC be the given triangle.

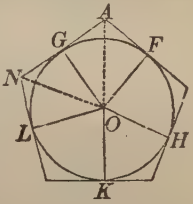


Bisect the base AB at D, and draw DH perpendicular to AB. Through C, the vertex of the triangle, draw CHG parallel to AB, and draw BG perpendicular to it: then will the rectangle DG be equivalent to the triangle ABC.

For, the triangle would be half a rectangle having the same base and altitude: hence, it is equivalent to DG, whose base is the half of AB, and altitude equal to that of the triangle.

PROBLEM X.

To inscribe a circle in a regular polygon.



Bisect any two sides of the polygon by the perpendiculars GO, FO, and with their point of intersection O, as a centre, and OG as a radius describe the circumference of a circle—this circle will touch all the sides of the polygon.

For, draw OA. Then in the two right angled triangles OAG and OAF, the side AO is common, and AG is equal to AF, since each is half of one of the equal sides of the polygon: hence, OG is equal to OF (Bk. I. Th. xix). In the same manner it may be shown that OH, OK and OL are all equal to each other: hence, a circle described with the centre O and radius OF will be inscribed in the polygon.

Cor. Hence, also the lines OA, ON &c., drawn to the angles of the polygon are equal.

APPENDIX

OF THE REGULAR POLYGONS.

1. In a regular polygon the angles are all equal to each other (Def. 3). If then, the sum of the inward angles of a regular polygon be divided by the number of angles, the quotient will be the value of one of the angles.

But the sum of the inward angles is equal to twice as many right angles, wanting four, as the polygon has sides, and we shall find the value in degrees by simply placing 90° for the right angle.

2. Thus, for the sum of all the angles of an equilateral triangle, we have

$$6\times90^\circ-4\times90^\circ=540^\circ-360^\circ=180^\circ$$

and for each angle

$$180^\circ\div3=60^\circ:$$

Hence, each angle of an equilateral triangle, is equal to 6 degrees.

3. For the sum of all the angles of a square, we have

$$8\times90^\circ-4\times90^\circ=720^\circ-360^\circ=360^\circ,$$

and for each of the angles

$$360^\circ\div4=90^\circ$$

4. For the sum of all the angles of a regular pentagon, we have

$$10\times90^\circ-4\times90^\circ=900^\circ-360^\circ=540^\circ,$$

and for each angle

$$540^\circ\div5=108^\circ.$$

5. For the sum of all the angles of a regular hexagon, we have

$$12\times90^\circ-4\times90^\circ=1080^\circ-360^\circ=720^\circ,$$

and of each angle

$$720^\circ\div6=120^\circ.$$

6. For the sum of the angles of a regular heptagon, we have

$$14\times90^\circ-4\times90^\circ=1260^\circ-360^\circ=900^\circ:$$

and for one of the angles

$$900^\circ\div7=128^\circ\ 34'+.$$

7. For the sum of the angles of a regular octagon, we have

$$16\times90^\circ-4\times90^\circ=1440^\circ-360^\circ=1080^\circ:$$

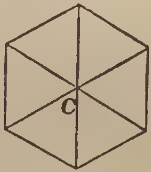
and for each angle

$$1080^\circ\div8=135^\circ$$

8. Since the sum of the angles about any point is equal to four right angles (Bk. I. Th. ii. Cor. 3), it may be observed that there are only three kinds of regular polygons, which can be arranged around any point, as *C*, so as exactly to fill up the space. These are,

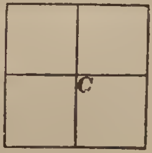
First.—Six equilateral triangles, in which each angle about *C* is equal to 60°, and their sum to

60° × 6 = 360.



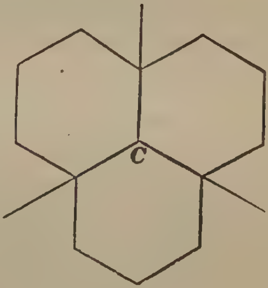
Second.- Four squares, in which each angle is equal to 90°, and their sum to

90° × 4 = 360°



Third.—Three hexagons, in which each angle is equal to 120, and the sum of the three to

120° × 3 = 360°.



GEOMETRY.

BOOK V.

OF PLANES AND THEIR ANGLES.

DEFINITIONS.

1. A straight line is *perpendicular to a plane*, when it is perpendicular to every straight line of the plane which it meets. The point at which the perpendicular meets the plane, is called the *foot* of the perpendicular.

2. If a straight line is perpendicular to a plane, the plane is also said to be perpendicular to the line.

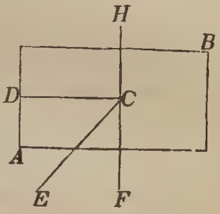
3. A line is parallel to a plane when it will not meet that plane, to whatever distance both may be produced. Conversely, the plane is then parallel to the line.

4. Two planes are parallel to each other, when they will not meet, to whatever distance both are produced.

5. If two planes are not parallel, they intersect each other in a line that is common to both planes: such line is called their *common intersection.*

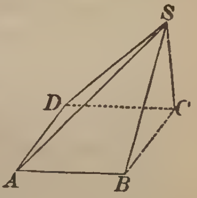
6. The space included between two planes is called a *diedral angle:* the planes are the *faces* of the angle, and their intersection the *edge.* A diedral angle is measured by two lines, one in each plane, and both perpendicular to the common intersection at the same point.

This angle may be acute, obtuse, or a right angle. When it is a right angle, the planes are said to be perpendicular to each other.



Let AB be a plane coinciding with the plane of the paper, and ECF a plane intersecting it in the line FH. Now, if from any point of the common intersection as C, we draw CD in the plane AB, and CE in the plane ECF, and both perpendicular to CF at C, then will the angle DCE measure the inclination between the two planes.

It should be remembered that the line EC is directly over the line CD.



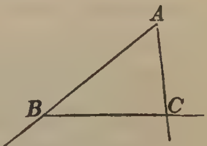
7. A polyedral angle is the angular space included between several planes meeting at the same point.

Thus, the polyedral angle S is formed by the meeting of the planes ASB, BSC, CSD, DSA.

8. The angle formed by three planes is called a *triedral angle.*

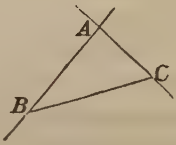
THEOREM I.

Two straight lines which intersect each other, lie in the same plane, and determine its position.



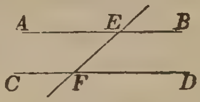
Let AB and AC be two straight lines which intersect each other at A.

Through AB conceive a plane to be passed, and let this plane be turned around AB until it embraces the point C: the plane will then contain the two lines AB, AC, and if it be turned either way it will depart from the point C, and consequently from the line AC. Hence, the position of the plane is determined by the single condition of containing the two straight lines AB, AC.



Cor. 1. A triangle ABC, or three points A, B, C, not in a straight line, determine the position of a plane.

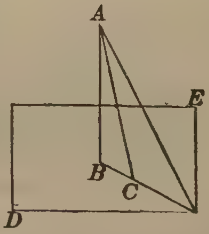
Cor. 2. Hence, also, two parallels AB, CD determine the position of a plane. For drawing EF, we see that the plane of the two straight lines AE, EF is that of the parallels AB, CD.



THEOREM II.

A perpendicular is the shortest line which can be drawn from a point to a plane.

Let A be a point above the plane DE, and AB a line drawn perpendicular to the plane: then will AB be shorter than any oblique line AC.



For, through B, the foot of the perpendicular, draw BC to the point where the oblique line AC meets the plane.

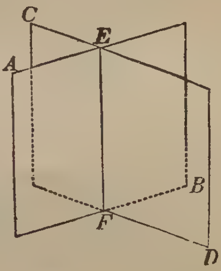
Now, since AB is perpendicular to the plane, the angle ABC will be a right angle (Def. 1.), and consequently less than the angle C: therefore, AB, opposite the angle C, will be less than AC opposite the angle B (Bk. I. Th. xi).

Cor. It is evident that if several lines be drawn from the point *A* to the plane, that those which are nearest the perpendicular *AB*, will be less than those more remote.

Sch. The distance from a point to a plane is measured on the perpendicular: hence, when the *distance* only is named, the shortest distance is always understood.

THEOREM III.

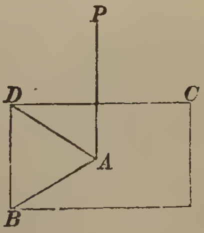
The common intersection of two planes is a straight line.



Let the two planes *AB*, *CD*, cut each other. Join any two points *E* and *F*, in the common intersection, by the straight line *EF*. This line will lie wholly in the plane *AB*, and also wholly in the plane *CD* (Bk. I. Def. 7); therefore, it will be in both planes at once, and consequently, is their common intersection.

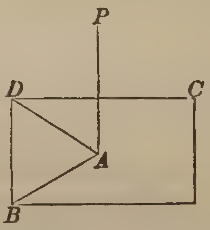
THEOREM IV.

A straight line which is perpendicular to two straight lines at their point of intersection, will be perpendicular to the plane of those lines.



Let the line *PA* be perpendicular to the two lines *AD*, *AB*: then will it be perpendicular to the plane *BC* which contains them.

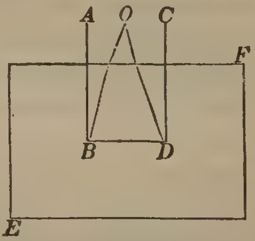
For, if *AP* is not perpendicular to the plane *BC*, suppose a plane to be drawn through A, that shall be perpendicular to AP.



Now, every line drawn through A, and perpendicular to AP, will be a line of this last plane (Def. 1): hence, this last plane will contain the lines AB, AD, and consequently, a line which is perpendicular to two lines at the point of intersection, will be perpendicular to the plane of those lines.

THEOREM V.

If two straight lines are perpendicular to the same plane they will be parallel to each other.



Let the two lines AB, CD, be perpendicular to the plane EF: then will they be parallel to each other.

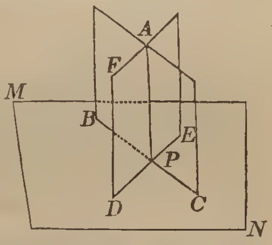
For, join B and D, the points in which the lines meet the plane EF.

Then, because the lines AB, CD, are perpendicular to the plane EF, they will be perpendicular to the line BD (Def. 1). Now, if BA and DC are not parallel, they will meet at some point as O: then, the triangle OBD would have two right angles, which is impossible (Bk. I. Th. xvii. Cor. 4).

Cor. If two lines are parallel, and one of them is perpendicular to a plane, the other will also be perpendicular to the same plane.

THEOREM VI.

If two planes intersect each other at right angles, and a line be drawn in one plane perpendicular to the common intersection, this line will be perpendicular to the other plane.

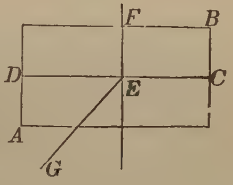


Let the plane FE be perpendicular to MN, and AP be drawn in the plane FE, and perpendicular to the common intersection DE: then will AP be perpendicular to the plane MN.

For, in the plane MN draw CP perpendicular to the common intersection DE. Then, because the planes MN and FE are perpendicular to each other, the angle APC, which measures their inclination, will be a right angle (Def. 6). Therefore, the line AP is perpendicular to the two straight lines PC and PD; hence, it is perpendicular to their plane MN (Th. iv).

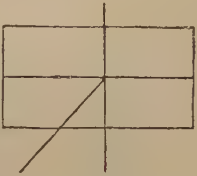
THEOREM VII.*

If one plane intersect another plane, the sum of the angles on the same side will be equal to two right angles.



Let the plane GEF intersect the plane AB in the line FE: then will the sum of the two angles on the same side be equal to two right angles.

For, from any point, as E, in the common intersection, draw the lines EG and DEC, one in each plane, and both perpendicular to the common intersection at E. Then, the line GE makes, with the line DEC, two angles, which together are equal to two right angles (Bk I. Th. ii): but these angles measure the inclination of the planes; therefore, the sum of the angles on the same side, which two planes make with each other, is equal to two right angles.



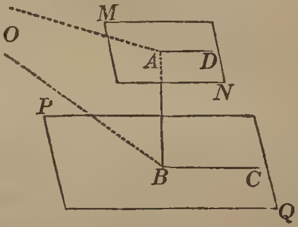
Cor. In like manner it may be demonstrated, that planes which intersect each other have their vertical or opposite angles equal.

THEOREM VIII.

Two planes which are perpendicular to the same straight line are parallel to each other.

Let the planes *MN* and *PQ* be perpendicular to the line *AB*: then will they be parallel.

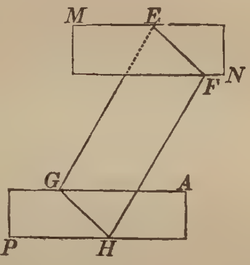
For, if they can meet any where, let *O* be one of their their common points, and draw *OB*, in the plane *PQ*, and *OA*, in the plane *MN*.



Now, since *AB* is perpendicular to both planes, it will be perpendicular to *OB* and *OA* (Def. 1): hence, the triangle *OAB* will have two right angles, which is impossible (Bk. I. Th. xvii. Cor. 4); therefore, the planes can have no point, as O, in common, and consequently, they are parallel (Def. 4).

THEOREM IX.

If a plane cuts two parallel planes, the lines of intersection will be parallel.

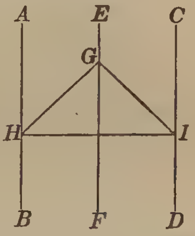


Let the parallel planes *MN* and *PA* be intersected by the plane *EH*: then will the lines of intersection *EF*, *GH*, be parallel.

For, if the lines *EF*, *GH*, were not parallel, they would meet each other if sufficiently produced, since they lie in the same plane. If this were so, the planes *MN*, *PA*, would meet each other, and, consequently, could not be parallel; which would be contrary to the supposition.

THEOREM X.

If two lines are parallel to a third line, though not in the same plane with it, they will be parallel to each other.



Let the lines *AB* and *CD* be each parallel to the third line *EF*, though not in the same plane with it: then will they be parallel to each other.

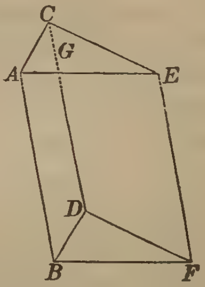
For, since *EF* and *CD* are parallel, they will lie in the same plane *FC* (Th. i. Cor. 2), and *AB*, *EF* will also lie in the plane *EB*.

At any point, *G*, in the line *EF*, let *GI* and *GH* be drawn in the planes *FC*, *BE*, and each perpendicular to *FE* at *G*.

Then, since the line *EF* is perpendicular to the lines *GH* *GI*, it will be perpendicular to the plane *HGI* (Th. iv). And since *FE* is perpendicular to the plane *HGI*, its parallels *AB* and *DC* will also be perpendicular to the same plane (Th. v). Hence, since the two lines *AB*, *CD*, are both perpendicular to the plane *HGI*, they will be parallel to each other

THEOREM XI.

If two angles, not situated in the same plane, have their sides parallel and lying in the same direction, the angles will be equal.



Let the angles ACE and BDF have the sides AC parallel to BD, and CE to DF: then will the angle ACE be equal to the angle BDF.

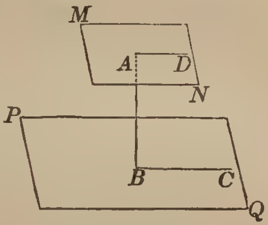
For, make AC equal to BD, and CE equal to DF, and join AB, CD, and EF; also, draw AE, BF.

Now since AC is equal and parallel to BD, the figure AD will be a parallelogram (Bk. I. Th. xxv); therefore, AB is equal and parallel to CD.

Again, since CE is equal and parallel to DF, CF will be a parallelogram, and EF will be equal and parallel to CD. Then, since AB and EF are both parallel to CD, they will be parallel to each other (Th. x); and since they are each equal to CD, they will be equal to each other. Hence, the figure $BAEF$ is a parallelogram (Bk. I. Th. xxv), and consequently, AE is equal to BF. Hence, the two triangles ACE and BDF have the three sides of the one equal to the three sides of the other, each to each, and therefore the angle ACE is equal to the angle BDF (Bk. I. Th. viii).

THEOREM XII.

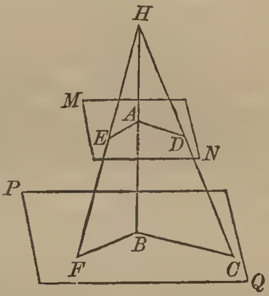
If two planes are parallel, a straight line which is perpendicular to the one will also be perpendicular to the other.



Let *MN* and *PQ* be two parallel planes, and let *AB* be perpendicular to *MN:* then will it be perpendicular to *PQ*.

For, draw any line, *BC*, in the plane *PQ*, and through the lines *AB*, *BC*, suppose the plane *ABC* to be drawn, intersecting the plane *MN* in the line *AD:* then, the intersection *AD* will be parallel to *BC* (Th. ix). But since *AB* is perpendicular to the plane *NM*, it will be perpendicular to the straight line *AD*, and consequently, to its parallel *BC* (Bk. I. Th. xii. Cor.)

In like manner, *AB* might be proved perpendicular to any other line of the plane *PQ*, which should pass through *B;* hence, it is perpendicular to the plane (Def. 1).



Cor. If from any point as *H*, any oblique lines, as *HEF*, *HDC*, be drawn, the parallel planes will cut these lines proportionally.

For, draw *HAB* perpendicular to the plane *MN:* then, by the theorem, it will also be perpendicular to *PQ*. Then draw *AD*, *AE*, *BC*, *BF*. Now, since *AE*, *BF*, are the intersections of the plane *FHB*, with the two parallel planes *MN*, *PQ*, they are parallel (Th. ix.); and so also are *AD*, *BC*.

Then, $HA : HB : HE : HF$,

and $HA : HB : HD : HC$,

hence, $HE : HF :: HD : HC$

GEOMETRY.

BOOK VI.

OF SOLIDS.

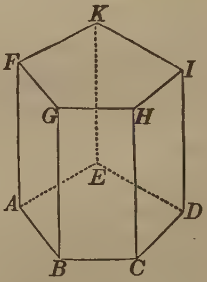
DEFINITIONS.

1. Every solid bounded by planes is called a *polyedron.*

2. The planes which bound a polyedron are called *faces.* The straight lines in which the faces intersect each other, are called the *edges* of the polyedron, and the points at which the edges intersect, are called the *vertices* of the angles, or vertices of the polyedron.

3. Two polyedrons are similar, when they are contained by the same number of similar planes, and have their polyedral angles equal, each to each.

4. A prism is a solid, whose ends are equal polygons, and whose side faces are parallelograms.

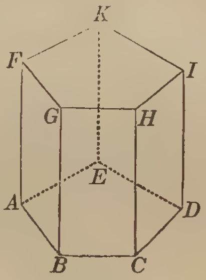


Thus, the prism whose lower base is the pentagon $ABCDE$, terminates in an equal and parallel pentagon $FGHIK$, which is called the *upper base.* The side faces of the prism are the parallelograms DH, DK, EF, AG, and BH. These are called the *convex*, or *lateral* surface of the prism

Of the Prism.

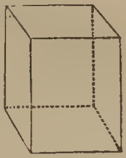
5. The altitude of a prism is the distance between its upper and lower bases: that is, it is a line drawn from a point of the upper base, perpendicular, to the lower base

6, A right prism is one in which the edges AF, BG, EK, HC, and DI, are perpendicular to the bases. In the right prism, either of the perpendicular edges is equal to the altitude. In the oblique prism the altitude is less than the edge.

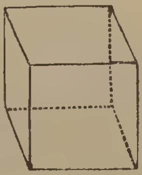


7. A prism whose base is a triangle, is called a *triangular* prism; if the base is a quadrangle, it is called a quadrangular prism; if a pentagon, a pentagonal prism; if a hexagon a hexagonal prism; &c.

8. A prism whose base is a parallelogram, and all of whose faces are also parallelograms, is called a parallelopipedon. If all the faces are rectangles, it is called a rectangular parallelopipedon.



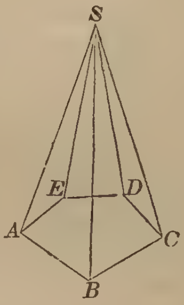
9. If the faces of the rectangular parallelopipedon are squares, the solid is called a *cube:* hence, the cube is a prism bounded by six equal squares



Of the Pyramid.

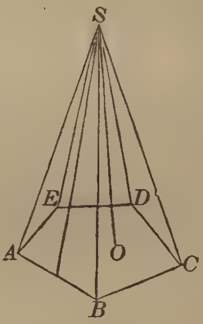
10. A pyramid is a solid, formed by several triangles united at the same point S, and terminating in the different sides of a polygon $ABCDE$.

The polygon $ABCDE$, is called the *base* of the pyramid; the point S, is called the *vertex*, and the triangles ASB, BSC, CSD, DSE, and ESA, form its *lateral*, or *convex* surface.

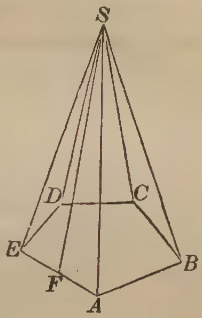


11. A pyramid whose base is a triangle, is called a *triangular* pyramid; if the base is a quadrangle, it is called a quadrangular pyramid; if a pentagon, it is called a petagonal pyramid; if the base is a hexagon, it is called a hexagonal pyramid; &c.

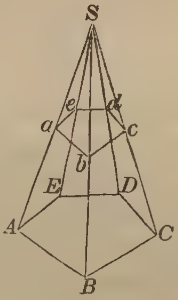
12. The *altitude* of a pyramid, is the perpendicular let fall from the vertex, upon the plane of the base. Thus, SO is the altitude of the pyramid S—$ABCDE$.



13. When the base of a pyramid is a regular polygon, and the perpendicular SO passes through the middle point of the base, the pyramid is called a right pyramid, and the line SO is called the *axis*.

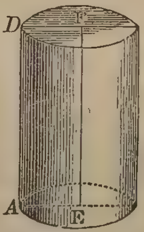


14. The *slant height* of a right pyramid, is a line drawn from the vertex, perpendicular to one of the sides of the polygon which forms its base. Thus. *SF* is the slant height of the pyramid *S—ABCDE*.



15. If from the pyramid *S—ABCDE* the pyramid S—*abcde* be cut off by a plane parallel to the base, the remaining solid, below the plane, is called the *frustum* of a pyramid.

The altitude of a frustum is the perpendicular distance between the upper and lower planes.



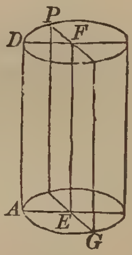
16. A *Cylinder* is a solid, described by the revolution of a rectangle, *AEFD*, about a fixed side, *EF*.

As the rectangle *AEFD*, turns around the side *EF*, like a door upon its hinges, the lines *AE* and *FD* describe circles, and the line *AD* describes the *convex surface* of the cylinder.

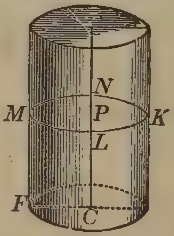
The circle described by the line *AE*, is called the *lower base* of the cylinder, and the circle described by *DF*, is called the *upper base*.

The immovable line EF is called the axis of the cylinder A cylinder, therefore, is a round body with circular ends

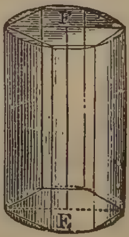
17. If a plane be passed through the axis of a cylinder, it will intersect the cylinder in a rectangle, PG, which is double the revolving rectangle DE.



18. If a cylinder be cut by a plane parallel to the base, the section will be a circle equal to the base. For, while the side FC, of the rectangle MC, describes the lower base, the equal side MP, will describe the circle $MLKN$, equal to the base.

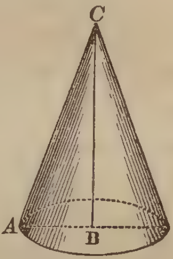


19. If a polygon be inscribed in the lower base of a cylinder, and a corresponding polygon be inscribed in the upper base, and their vertices be joined by straight lines, the prism thus formed is said to be *inscribed* in the cylinder.



Of the Cone.

20. A *cone* is a solid, described by the revolution of a right angled triangle, ABC, about one of its sides, CB.

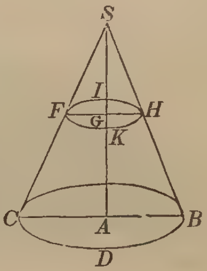


The circle described by the revolving side, AB, is called the *base* of the cone.

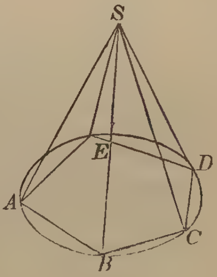
The hypothenuse, AC, is called the *slant height* of the cone, and the surface described by it, is called the *convex surface* of the cone.

The side of the triangle, CB, which remains fixed, is called the *axis*, or *altitude* of the cone, and the point C, the *vertex* of the cone.

21. If a cone be cut by a plane parallel to the base, the section will be a circle. For, while in the revolution of the right angled triangle SAC, the line CA describes the base of the cone, its parallel FG will describe a circle $FKHI$, parallel to the base. If from the cone S—CDB, the cone S—FKH be taken away, the remaining part is called the *frustum* of the cone.



22. If a polygon be inscribed in the base of a cone, and straight lines be drawn from its vertices to the vertex of the cone, the pyramid thus formed is said to be inscribed in the cone. Thus, the pyramid S—$ABCD$ is inscribed in the cone.

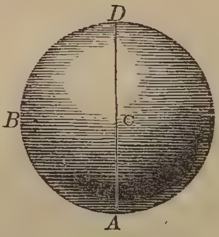


23. Two cylinders are similar, when the diameters of their bases are proportional to their altitudes.

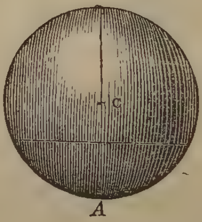
24. Two cones are also similar, when the diameters of their bases are proportional to their altitudes.

25. A *sphere* is a solid terminated by a curved surface, all the points of which are equally distant from a certain point within called the centre.

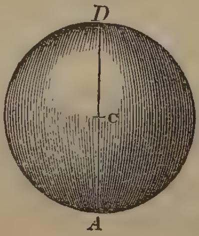
26. The sphere may be described by revolving a semicircle, *ABD*, about the diameter *AD*. The plane will describe the solid sphere, and the semicircumference *ABD* will describe the surface.



27. The *radius* of a sphere is a line drawn from the centre to any point of the circumference. Thus, *CA* is a radius.



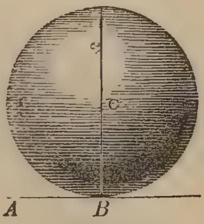
28. The *diameter* of a sphere is a line passing through the centre, and terminated by the circumference. Thus, *AD* is a diameter.



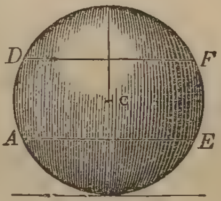
29. All diameters of a sphere are equal to each other; and each is double a radius.

30. The axis of a sphere is any line about which it revolves; and the points at which the axis meets the surface, are called the *poles*.

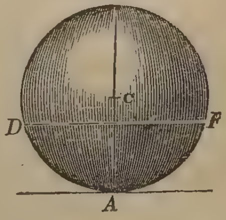
31. A plane is *tangent* to a sphere when it has but one point in common with it. Thus, *AB* is a tangent plane, touching the sphere at *B*.



32. A *zone* is a portion of the surface of a sphere, included between two parallel planes which form its bases. Thus, the part of the surface included between the planes *AE* and *DF* is a zone. The bases of this zone are the two circles whose diameters are *AE* and *DF*.



33. One of the planes which bound a zone may become tangent to the sphere; in which case the zone will have but one base. Thus, if one plane be tangent to the sphere at *A*, and another plane cut it in the circle *DF*, the zone included between them, will have but one base.

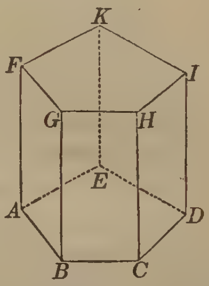


34. A *spherical segment* is a portion of the solid sphere included between two parallel planes. These parallel planes are its bases. If one of the planes is tangent to the sphere, the segment will have but one base.

35. The *altitude* of a zone or segment, is the distance between the parallel planes which form its bases.

THEOREM I.

The convex surface of a right prism is equal to the perimeter of its base multiplied by its altitude.



Let $ABCDE$—K be a right prism: then will its convex surface be equal to

$$(AB+BC+CD+DE+EA)\times AF.$$

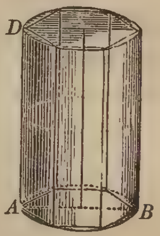
For, the convex surface is equal to the sum of the rectangles AG, BH, CI, DK, and EF, which compose it; and the area of each rectangle is equal to the product of its base by its altitude. But the altitude of each rectangle is equal to the altitude of the prism: hence, their areas, that is, the convex surface of the prism, is equal to

$$(AB+BC+CD+DE+EA)\times AF;$$

that is, equal to the perimeter of the base of the prism multiplied by its altitude.

THEOREM II.

The convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude

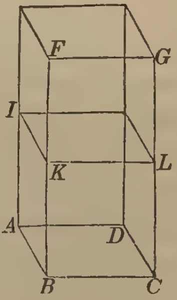


Let DB be a cylinder, and AB the diameter of its base: the convex surface will then be equal to the altitude AD multiplied by the circumference of the base.

For, suppose a regular prism to be inscribed within the cylinder. Then, the convex surface of the prism will be equal to the perimeter of the base multiplied by the altitude (Th. i). But the altitude of the prism is the same as that of the cylinder; and if we suppose the sides of the polygon, which forms the base of the prism, to be indefinitely increased, the polygon will become the circle (Bk. IV. Th. xxiii. Sch.), in which case, its perimeter will become the circumference, and the prism will coincide with the cylinder. But its convex surface is still equal to the perimeter of its base multiplied by its altitude: hence, the convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude.

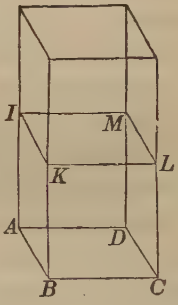
THEOREM III.

In every prism the sections formed by planes parallel to the base are equal polygons.



Let AG be any prism, and IL a section made by a plane parallel to the base AC: then will the polygon IL be equal to AC.

For, the two planes AC, IL, being parallel, the lines AB, IK, in which they intersect the plane AF, will also be parallel (Bk. V. Th. ix). For a like reason, BC and KL will be par-



allel; also, CD will be parallel to LM, and AD to IM.

But, since AI and BK are parallel, the figure AK is a parallelogram: hence AB is equal to IK (Bk. I. Th. xxiii). In the same way it may be shown that BC is equal to KL, CD to LM, and AD to IM.

But, since the sides of the polygon AC are respectively parallel to the sides of the polygon IL, it follows that their corresponding angles are equal (Bk. V. Th. xi), viz., the angle A to the angle I, the angle B to K, the angle C to L, and the angle M to D; hence, the polygon IL is equal to AC.

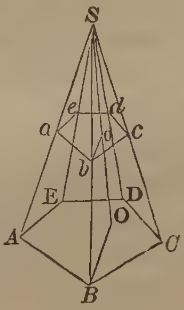
Sch. It was shown in Definition 18, that the section of a cylinder, by a plane parallel to the base, is a circle equal to the base.

THEOREM IV.

If a pyramid be cut by a plane parallel to the base,

I. *The edges and altitude will be divided proportionally.*

II. *The section will be a polygon similar to the base.*



Let the pyramid $S—ABCDE$, of which SO is the altitude, be cut by the plane $abcde$ parallel to the base: then will,

$$Sa : SA :: Sb : SB,$$

and the same for the other edges; and the polygon $abcde$ will be similar to the base $ABCDE$.

First. Since the planes ABC and abc

are parallel, their intersections, AB, ab, by the plane SAB, will also be parallel (Bk. V. Th. ix); hence, the triangles SAB, sab, are similar, and we have

$$SA : Sa :: SB : Sb;$$

for a similar reason, we have

$$SB : Sb :: SC : Sc;$$

and the same for the other edges: hence, the edges SA, SB, SC, &c., are cut proportionally at the points a, b, c, &c.

The altitude SO is likewise cut proportionally at the point

The altitude SO is likewise cut in the same proportion at the point o; for, since BO is parallel to bo, we have

$$SO : So :: SB : Sb.$$

Secondly. Since ab is parallel to AB, bc to BC, cd to CD &c.; the angle abc is equal to ABC, the angle bcd to BCD, and so on (Bk. V. Th. xi).

Also, by reason of the similar triangles, SAB, Sab, we have

$$AB : ab : SB : Sb,$$

and by reason of the similar triangles SBC, Sbc, we have

$$SB : Sb :: BC : bc;$$

hence (Bk. III. Th. v),

$$AB : ab :: BC : bc;$$

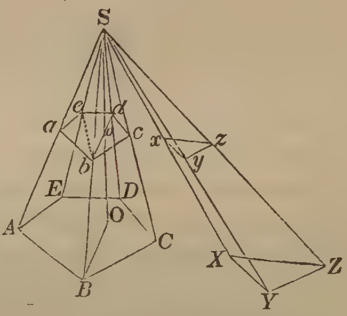
and for a similar reason, we also have

$$BC : bc :: CD : cd, \&c.$$

Hence, the polygons $ABCDE$, $abcde$, having their angles respectively equal, and their homologous sides proportional, are similar.

THEOREM V.

If two pyramids, having equal altitudes and their bases in the same plane, be intersected by planes parallel to the plane of the bases, the sections in each pyramid will be proportional to the bases.



Let S—$ABCDE$, and S—XYZ, be two pyramids, having a common vertex, and their bases situated in the same plane. If these pyramids are cut by a plane parallel to the plane of their bases, giving the sections $abcde$, xyz, then will the sections $abcde$, xyz, be to each other as the bases $ABCDE$, XYZ.

For, the polygons $ABCDE$, $abcde$, being similar, their surfaces are as the squares of the homologous sides AB, ab;

but $AB : ab :: SA : Sa$;

hence, $ABCDE : abcde :: \overline{SA}^2 : \overline{Sa}^2$

For the same reason,

$$XYZ : xyz :: \overline{SX}^2 : \overline{Sx}^2.$$

But since abc and xyz are in one plane, the lines SA, Sa, SX, Sx, are proportional to SO, So: (Bk. V. Th xii. Cor.), therefore,

$$SA : Sa :: SX : Sx:$$

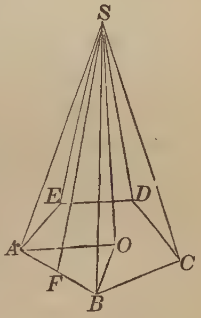
hence, $ABCDE : abcde :: XYZ : xyz$.

consequently, the sections $abcde$, xyz, are to each other as the bases $ABCDE$, XYZ.

Cor. If the bases $ABCDE$, XYZ, are equivalent, any sections $abcde$, xyz, made at equal distances from the bases, will be also equivalent.

THEOREM VI.

The convex surface of a right pyramid is equal to half the product of the perimeter of its base multiplied by the slant height.



Let $S—ABCDE$ be a right pyramid, SF its slant height: then will its convex surface be equal to half the product

$$SF\times(AB+BC+CD+DE+EA).$$

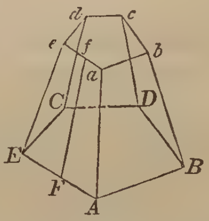
For, since the pyramid is right, the point O, in which the axis meets the base, is the centre of the polygon $ABCDE$; hence, the lines OA, OB, &c. drawn to the vertices of the base, are equal (Bk. IV. prob. x. Cor).

Now, in the right angled triangles SAO, SBO, the bases and perpendiculars are equal: hence, the hypothenuses are equal; and in the same way it may be proved that all the edges of the pyramid are equal. The triangles, therefore, which form the convex surface of the prism, are all equal to each other.

But the area of either of these triangles, as SAB, is equal to half the product of the base AB, by the slant height of the pyramid SF: hence, the area of all the triangles, which form the convex surface of the pyramid, is equal to half the product of the perimeter of the base by the slant height.

THEOREM VII.

The convex surface of the frustum of a regular pyramid is equal to half the sum of the perimeters of the upper and lower bases multiplied by the slant height.



Let a—$ABCDE$ be the frustum of a regular pyramid: then will its convex surface be equal to half the product of the perimeter of its two bases multiplied by the slant height Ff.

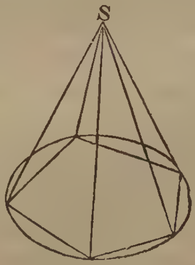
For, since the upper base $abcde$, is similar to the lower base $ABCDE$ (Th. iv), and since $ABCDE$ is a regular polygon, it follows that the sides ab, bc, cd, de, and ea, are all equal to each other.

Hence, the trapezoids $EAae$, $ABba$, &c., which form the convex surface of the frustum are equal. But the perpendicular distance between the parallel sides of these trapezoids is equal to Ef, the slant height of the frustum.

Now, the area of either of the trapezoids, as $AEea$, is equal to half the product of $Ff \times (EA + ea)$ (Bk. IV. Th. x): hence, the area of all of them, that is, the convex surface of the frustum, is equal to half the sum of the perimeters of the upper and lower bases, multiplied by the slant height.

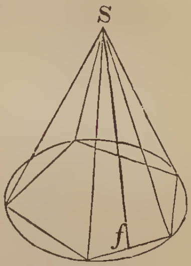
THEOREM VIII.

The convex surface of a cone is equal to half the product of the circumference of the base multiplied by the slant height.



In the circle which forms the base of the cone, inscribe a regular polygon, and join the vertices with the vertex S, of the cone We shall then have a right pyramid inscribed in the cone.

The convex surface of this pyramid will be equal to half the product of the perimeter of the base by the slant height (Th. vi).

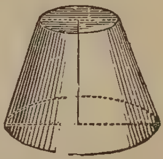


Let us now suppose the number of sides of the polygon to be indefinitely increased: the polygon will then coincide with the base of the cone, the pyramid will become the cone, and the line Sf which measures the slant height of the pyramid, will then measure the slant height of the cone.

Hence, the convex surface of the cone is equal to half the product of the slant height by the circumference of the base.

THEOREM IX.

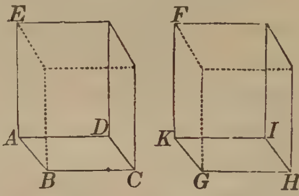
The convex surface of the frustum of a cone is equal to half the sum of the circumferences of its two bases multiplied by the slant height.



For, if we suppose the frustum of a right pyramid to be inscribed in the frustum of a cone, its convex surface will be equal to half the product of its slant height by the perimeters of its two bases. But if we increase the number of sides of the polygon indefinitely, the frustum of the pyramid will become the frustum of the cone: hence, the area of the frustum of the cone is equal to half the sum of the circumferences of its two bases multiplied by the slant height.

THEOREM X.

Two rectangular parallelopipedons, having equal altitudes and equal bases, are equal.



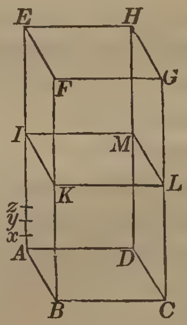
Let E—$ABCD$, and F—$KGHI$, be two rectangular parallelopipedons having equal bases, AC and KH, and equal altitudes, AE and KF: then will they be equal.

For, apply the base of the one parallelopipedon to that of the other, and since the bases are equal, they will coincide

Again, since the edges are perpendicular to the bases, the edges of the one parallelopipedon will coincide with those of the other; and since the altitude AE is equal to KF, the planes of the upper bases will coincide. Hence, the parallelopipedons will coincide, and consequently they are equal.

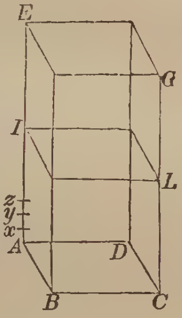
THEOREM XI.

Two rectangular parallelopipedons, which have the same base, are to each other as their altitudes.



Let the parallelopipedons AG, AL, have the same base BD, then will they be to each other as their altitudes AE AI.

Suppose the altitudes AE, AI, to be to each other as two whole numbers, as 15 is to 8, for example. Divide AE into 15 equal parts, whereof AI will contain 8; and through x. y, z, &c., the points of division, draw planes

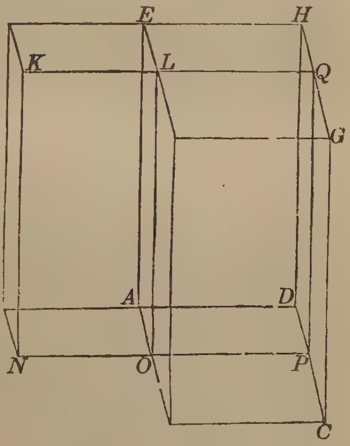


parallel to the base. These planes will cut the solid *AG* into 15 partial parallelopipedons, all equal to each other, because they have equal bases and equal altitudes—equal bases, since every section, *IL*, made parallel to the base *BD*, of a prism, is equal to that base; equal altitudes, because the altitudes are the equal divisions *Ax*, *xy*, *yz*, &c. But of these 15 equal parallelopipedons, 8 are contained in *AL*; hence, *solid AG* : *solid AL* :: 15 : 8 or generally,

solid AG : *solid AL* :: *AE* : *AI*.

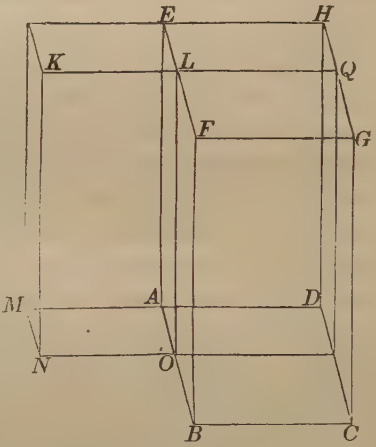
THEOREM XII.

Two regular parallelopipedons, having the same altitude, are to each other as their bases.



Let the parallelopipedons *AG*, *AK*, have the same altitude *AE*; then will they be to each other as their bases *AC*, *AN*.

Having placed the two solids by the side of each other, as the figure represents, produce the plane *ONKL* until it meets the plane *DCGH* in *PQ*; you will thus



have a third parallelopipedon AQ, which may be compared with each of the parallelopipedons AG, AK. The two solids AG, AQ, having the same base $AEHD$, are to each other as their altitudes AB, AO; in like manner, the two solids AQ AK, having the same base $AOLE$, are to each other as their altitudes AD, AM.

Hence, we have the two proportions,

$$\text{solid } AG : \text{solid } AQ :: AB : AO,$$
$$\text{solid } AQ : \text{solid } AK :: AD : AM.$$

Multiplying together the corresponding terms of these proportions, and omitting the common multiplier *solid* AQ, we have

$$\text{solid } AG : \text{solid } AK :: AB \times AD : AO \times AM.$$

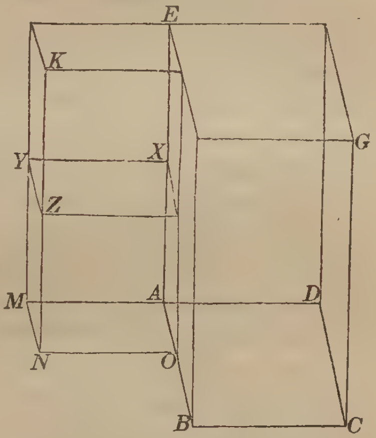
But $AB \times AD$ represents the base $ABCD$; and $AO \times AM$ represents the base $AMNO$: hence, two rectangular parallelopipedons of the same altitude are to each other as their bases.

THEOREM XIII.

Any two rectangular parallelopidedons are to each other as the products of their three dimensions.

For, having placed the two solids AG, AZ, (see next figure) so that their surfaces have the common angle BAE, produce the planes necessary for completing the third parallelopipedon AK, having the same altitude with the parallelopipedon AG. By the last proposition we shall have the proportion, *solid AG* : *solid AK* : : *ABCD* : *AMNO*

But the two parallelopipedons *AK*, *AZ*, having the same base *AMNO*, are to each other as their altitudes *AE*, *AX*; hence, we have



solid AK : *solid AZ* : : *AE* : *AX*.

Multiplying together the corresponding terms of these proportions, and omitting in the result the common multiplier *solid AK*, we shall have

solid AG : *solid AZ* : : $ABCD \times AE$: $AMNO \times AX$.

Instead of the bases *ABCD* and *AMNO*, put $AB \times AD$ and $AO \times AM$, and we have

solid AG : *solid AZ* : : $AB \times AD \times AE$: $AO \times AM \times AX$.

Hence, any two rectangular parallelopipedons are to each other as the product of their three dimensions.

Sch. We are consequently authorized to assume, as the measure of a rectangular parallelopipedon, the product of its three dimensions.

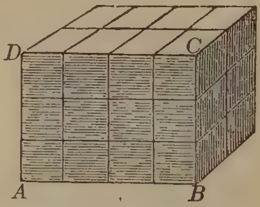
In order to comprehend the nature of this measurement, it is necessary to reflect, that the number of linear units in one dimension of the base multiplied by the number of linear units of the other dimension of the base, will give the number of superficial units in the base of the parallelopipedon (Bk. IV Th. vi. Sch). For each unit in height, there are evidently as many solid units as there are superficial units in the base. Therefore, the product of the number of superficial units in the base multiplied by the number of linear units in the altitude is the number of solid units in the parallelopipedon.

If the three dimensions of another parallelopipedon are valued according to the same linear unit, and multiplied together in the same manner, the two products will be to each other as the solids, and will serve to express their relative magnitude.

Let us illustrate this by an example.

Let *ABCD* be the base of a parallelopipedon, and suppose $AB=4$ feet, and $BC=3$ feet. Then the number of square feet in the base *ABCD* will be equal to $3\times4=12$ square feet

Therefore, 12 equal cubes of 1 foot each, may be placed by the side of each other on the base. If the parallelopipedon be 1 foot in height, it will contain 12 cubic feet; were it 2 feet in height, it would contain two tiers of cubes, or 24 cubic feet; were it 3 feet in height, it would contain three tiers of cubes, or 36 cubic feet.



The magnitude of a solid, its volume or extent, forms what is called its *solidity*; and this word is exclusively employed to designate the measure of a solid; thus, we say the solidity of a rectangular parallelopipedon is equal to the product of its base by its altitude, or to the product of its three dimensions

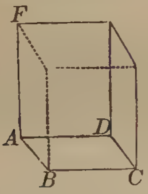
Of Parallelopipedons.

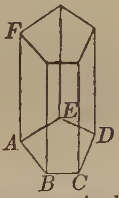
As the cube has all its three dimensions equal, if the side is 1, the solidity will be $1 \times 1 \times 1 = 1$; if the side is 2, the solidity will be $2 \times 2 \times 2 = 8$; if the side is 3, the solidity will be $3 \times 3 \times 3 = 27$; and so on: hence, if the sides of a series of cubes are to each other as the numbers 1, 2, 3, &c. the cubes themselves, or their solidities, will be as the numbers 1, 8, 27, &c. Hence it is, that in arithmetic, the *cube* of a number is the name given to a product which results from three factors, each equal to this number.

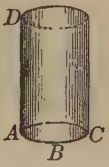
THEOREM XIV.

If a parallelopipedon, a prism, and a cylinder, have equivalent bases and equal altitudes, they will be equivalent.

Let *F—ABCD*, be a parallelopipedon; *F—ABCDE*, a prism; and *D—ABC*, a cylinder, having equivalent bases and equal altitudes: then will they be equivalent.





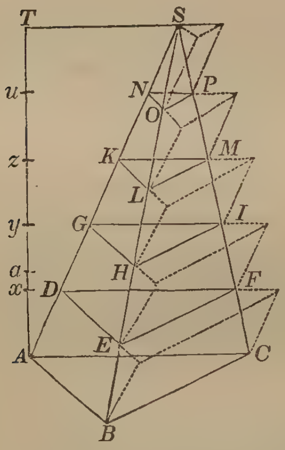


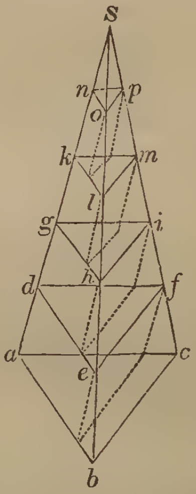
For, since their bases are equivalent they will contain the same number of units of surface (Bk. IV. Def. 9). Now, for each unit of height there will be one tier of equal cubes in each solid, and since the altitudes are equal, the number of tiers in each solid will be equal: hence, the solidities will be equal, and therefore the solids will be equivalent.

Cor Hence, we conclude, that the solidity of a prism or cylinder is equal to the area of its base multiplied by its altitude.

THEOREM XV.

Two triangular pyramids, having equivalent bases and equal altitudes, are equivalent, or equal in solidity.





Let their equivalent bases, ABC, abc, be situated in the same plane, and let AT be their common altitude. If they are not equivalent, let S—abc be the smaller; and suppose Aa to be the altitude of a prism, which, having ABC for its base, is equal to their difference.

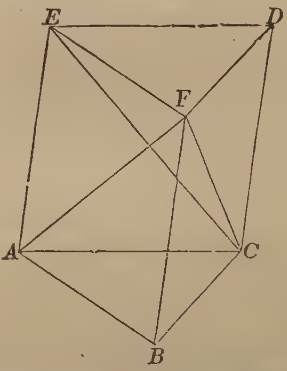
Divide the altitude AT into equal parts Ax, xy, yz, &c., each less than Aa, and let k be one of those parts: through the points of division pass planes parallel to the plane of the bases: the corresponding sections formed by these planes in the two pyramids will be respectively equivalent, namely, DEF to def, GHI to ghi, &c. (Th. v. Cor.)

This being granted, upon the triangles ABC, DEF, GHI, &c., taken as bases, construct exterior prisms having for edges the parts AD, DG, GK, &c., of the edge SA; in like manner, on bases def, ghi, klm, &c., in the second pyramid, construct interior prisms, having for edges the corresponding parts of Sa. It is plain that the sum of the exterior prisms of the pyramid $S—ABC$ will be greater than the pyramid; while the sum of the interior prisms of the pyramid $S—abc$, will be less than the pyramid. Hence, the difference between these sums will be greater than the difference between the pyramids.

Now, beginning with the bases ABC, abc, the second exterior prism $EFD—G$ is equivalent to the first interior prism $efd—a$, because they have the same altitude k, and their bases DEF, def, are equivalent; for like reasons, the third exterior prism $HIG—K$, and the second interior prism $hig—d$, are equivalent; the fourth exterior and the third interior; and so on, to the last of each series. Hence, all the exterior prisms of the pyramid $S—ABC$, excepting the first prism $BCA—D$, have equivalent corresponding ones in the interior prisms of the pyramid $S—abc$: hence, the prism $BCA—D$ is the difference between the sum of all the exterior prisms of the pyramid $S—ABC$, and of the interior prisms of the pyramid $S—abc$ But this difference has already been proved to be greater than that of the two pyramids: which, by supposition, differ by the prism $a—ABC$: hence, the prism $BCA—D$, must be greater than the prism $a—ABC$. But in reality it is less, for they have the same base ABC, and the altitude Ax, of the first, is less than Aa, the altitude of the second. Hence, the supposed inequality between the two pyramids cannot exist; hence, the two pyramids; $S—ABC$, $S—abc$, having equal altitudes and equivalent bases, are themselves equivalent.

THEOREM XVI.

Every triangular pyramid is a third part of a triangular prism which has an equal base and the same altitude.



Let *F—ABC* be a triangular pyramid, *ABC—DEF* a triangular prism of the same base and the same altitude: the pyramid will be equal to a third of the prism.

Cut off the pyramid *F—ABC* from the prism, by the plane *FAC*; there will remain the solid *F—ACDE*, which may be considered as a quadrangular pyramid, whose vertex is *F*, and whose base is the parallelogram *ACDE*. Draw the diagonal *CE*, and pass the plane *FCE*, which will cut the quadrangular pyramid into two triangular ones, *F–ACE*, *F–CDE*. These two triangular pyramids have for their common altitude the perpendicular let fall from *F* on the plane *ACDE;* and their bases are also equal, being halves of the parallelogram *AD:* hence, the pyramid *F–ACE*, and the pyramid *F–CDE*, are equivalent (Th. xv).

But the pyramid *F—CDE*, and the pyramid *F—ABC*, have equal bases, *ABC*, *DEF;* they have also the same altitude, namely, the distance between the parallel planes *ABC*, *DEF*, hence, the two pyramids are equivalent. Now, the pyramid *F—CDE* has already been proved equivalent to *F—ACE*; hence, the three pyramids *F—ABC*, *F—CDE*, *F—ACE*, which compose the prism *ABC—DEF* are all equivalent

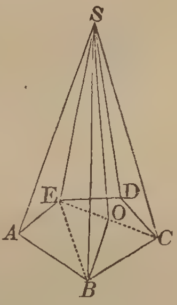
Hence, the pyramid *F—ABC* is the third part of the prism *ABC—DEF*, which has an equal base and the same altitude.

Cor. The solidity of a triangular pyramid is equal to a third part of the product of its base by its altitude.

THEOREM XVII.

The solidity of every pyramid is equal to the base multiplied by a third of the altitude.

Let *S—ABCDE* be a pyramid.



Pass the planes *SEB*, *SEC* through the diagonals *EB*, *EC*; the polygonal pyramid *S—ABCDE* will be divided into several triangular pyramids all having the same altitude *SO*. But each of these pyramids is measured by multiplying its base *ABE*, *BCE*, or *CDE*, by the third part of its altitude *SO* (Th. xvi. Cor.); hence the sum of these triangular pyramids, or the polygonal pyramid *S—ABCDE*, will be measured by the sum of the triangles *ABE*, *BCE*, *CDE*, or the polygon *ABCDE*, multiplied by one third of *SO*.

Cor. 1. Every pyramid is the third part of the prism which has the same base and the same altitude.

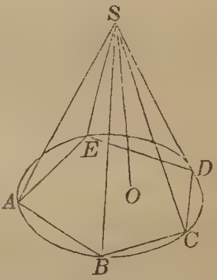
Cor. 2. Two pyramids having the same altitude, are to each other as their bases.

Cor. 3. Two pyramids having equivalent bases, are to each other as their altitudes.

Cor. 4. Pyramids are to each other as the products of their bases by their altitudes.

THEOREM XVIII.

The solidity of a cone is equal to one third of the product of the base multiplied by the altitude.



Let $ABCDE$ be the base, S the vertex, and SO the altitude of the cone: then will its solidity be equal to one third the product of its base by its altitude SO.

Inscribe in the base of the cone any regular polygon, $ABCDE$, and join the vertices A, B, C, &c., with the vertex S, of the cone; then will there be inscribed in the cone a right pyramid, having for its base the polygon $ABCDE$. The solidity of this pyramid is equal to one third of the base multiplied by the altitude (Th. xvii).

Let now, the number of sides of the polygon be indefinitely increased: the polygon will then become equal to the circle, and the pyramid and cone will coincide and become equal. But the solidity of the pyramid will still be equal to one third of the product of the base multiplied by the altitude, whatever be the number of sides of the polygon which forms its base; hence, the solidity of the cone is equal to one third of the product of its base multiplied by its altitude.

Cor. 1. A cone is the third part of a cylinder having the same base and the same altitude; whence it follows:

1st, That cones of equal altitudes are to each other as their bases.

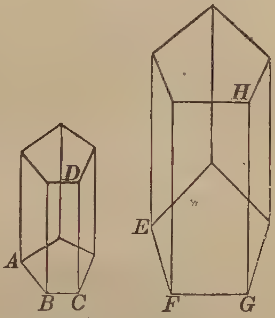
2nd, That cones of equal bases are to each other as their altitudes.

Cor. 2. The solidity of a cone is equivalent to the solidity of a pyramid having an equivalent base and the same altitude.

THEOREM XIX.

Similar prisms are to each other as the cubes of their homologous edges.

Let $ABC\text{--}D$, $EFG\text{--}H$ be similar prisms: then we shall have



$$\text{solid } AD : \text{solid } EH :: \overline{AB}^3 : \overline{EF}^3;$$

or $$\text{solid } AD : \text{solid } EH :: \overline{CD}^3 : \overline{HG}^3;$$

or, the solids will be to each other as the cubes of any other of their homologous edges.

For, the solids are to each other as the products of their bases and altitudes (Th. xiv. Cor.), that is,

$$\text{solid } ABC\text{–}D : \text{solid } EFG\text{–}H :: ABC \times CD : EFG \times GH.$$

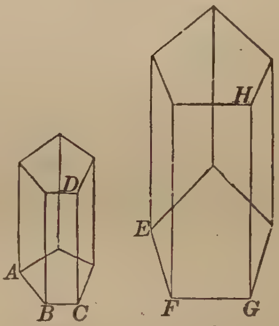
But the bases being similar polygons are to each other as the squares of their like sides (Bk. IV. Th. xxi); that is,

$$ABC : EFG :: \overline{AB}^2 : \overline{EF}^2,$$

therefore,

$$\text{solid } ABC\text{–}D : \text{solid } EFG\text{–}H :: \overline{AB}^2 \times CD : \overline{EF}^2 \times GH.$$

But since the solids are similar, the parallelograms *BD* and *FH* are similar (Def. 3): hence, *CD* and *GH* are proportional to *BC* and *FG*, and consequently to *AB* and *EF*: hence, we have,



$$solid\ ABC\text{–}D : solid\ EFG\text{–}H :: \overline{AB}^2 \times AB : \overline{EF}^2 \times EF.$$

that is,

$$solid\ ABC\text{–}D : solid\ EFG\text{–}H :: \overline{AB}^3 : \overline{EF}^3;$$

and in a similar manner it may be shown that the solids are to each other as the cubes of any other homologous edges.

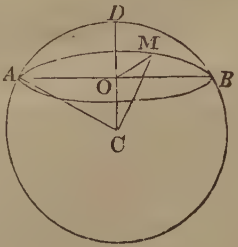
Cor. Since cylinders are to each other as the product of their bases and altitudes (Th. xiv. Cor.), it follows that similar cylinders are to each other as the cubes of the linear dimensions.

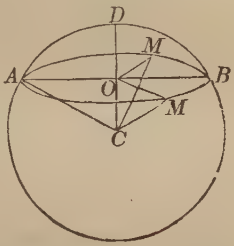
THEOREM XX.

Every section of a sphere, made by a plane, is a circle.

Let *AMB* be a section, made by a plane, in the sphere whose centre is *C*.

From the centre *C* draw *CO*, perpendicular to the plane *AMB*, and also draw the lines *CA*, *CM*, &c., to the points of the curve *AMB*, which terminate the section, and join *OA*, *OM*, &c.





Then, since *CO* is perdendicular to the plane *AMB*, the angles *COA*, *COM* &c., will be right angles, and since the radii of the sphere are all equal, the right angled triangles *CAO*, *COM*, &c., will have the hypothenuses equal, and the side *CO* common: hence, the remaining sides will be equal (Bk. I. Th. xix). Therefore, all lines drawn from *O* to any point of the curve *AMB* are equal: hence *AMB* is a circle.

Cor. 1. If the section passes through the centre of the sphere, its radius will be the radius of the sphere: hence, all great circles are equal.

Cor. 2. Two great circles always bisect each other; for their common intersection, passing through the centre, is a diameter.

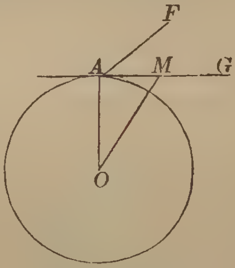
Cor. 3. Every great circle divides the sphere and its surface into two equal parts: for, if the two hemispheres were separated and afterwards placed on the common base, with their convexities turned the same way, the two surfaces would exactly coincide, no point of the one being nearer the centre than any point of the other.

Cor. 4. The centre of a small circle, and that of the sphere, are in the same straight line, perpendicular to the plane of the small circle.

Cor. 5. Small circles are the less the farther they lie from the centre of the sphere; for the greater CO is, the less is the chord AB, the diameter of the small circle AMB.

THEOREM XXI.

Every plane perpendicular to a radius at its extremity is tangent to the sphere.

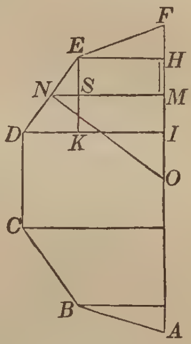


Let FAG be a plane perpendicular to the radius OA, at its extremity A. Any point M, in this plane, being assumed, and OM, AM, being drawn, the angle OAM will be a right angle, and hence, the distance OM will be greater than OA. Hence, the point M lies without the sphere; and as the same can be shown for every other point of the plane FAG, this plane can have no point but A common to it and the surface of the sphere; hence it is a tangent plane (Def. 31).

Sch. In the same way it may be shown, that two spheres have but one point in common, and therefore touch each other, when the distance between their centres is equal to the sum, or the difference of their radii; in either case, the centres and the point of contact lie in the same straight line.

THEOREM XXII.

If a regular semi-polygon be revolved about a line passing through the centre and the vertices of two opposite angles, the surface described by its perimeter will be equal to the axis multiplied by the circumference of the inscribed circle.



Suppose the regular semi-polygon *ABCDE* to be revolved about the line *AF* as an axis: then will the surface described by its perimeter be equal to *AF* multiplied by the circumference of the inscribed circle.

From *E* and *D*, the extremities of one of the equal sides, let fall the perpendiculars *EH*, *DI*, on the axis *AF*, and from the centre *O*, draw *ON* perpendicular to the side *DE*: *ON* will then be the radius of the inscribed circle (Bk. IV. Prob. x).

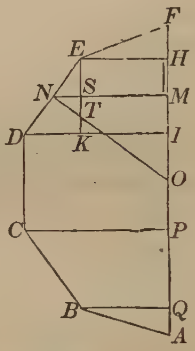
Let us first find the measure of the surface described by one of the equal sides, as *DE*.

From *N*, the middle point of *DE*, draw *NM* perpendicular to the axis *AF*, and through *E*, draw *EK*, parallel to it, meeting *MN* in *S*.

Then, since *EN* is half of *ED*, *NS* will be half of *DK* (Bk. IV. Th. xiii): and hence, *NM* is equal to half the sum of $EH+DI$.

But, since the circumferences of circles are to each other as their diameters (Bk. IV. Th. xxiv), or as their radii, the halves of the diameters, we shall have the circumference described by the point *N*, equal to half the sum of the circumferences described by the points *D* and *E*.

But in the revolution of the polygon the line *ED* describes the surface of the frustum of a cone, the measure of which is equal to *DE* multiplied into half the sum of the circumferences of the two bases (Th. ix); that is, equal to *DE* into the circumference described by the point *N*.



But, the triangle *ENS* is similar to *SNT* (Bk. IV. Th. xviii), and also to *EDK*, and since *TNS* is similar to *ONM*, it follows that *EDK* and *ONM* are similar; hence,

$$ED : EK \text{ or } HI :: ON : NM,$$

or $ED : HI :: \textit{circumference } ON : \textit{circumference } MN.$

consequently,

$$ED \times \textit{circumference } MN = HI \times \textit{circumference } ON,$$

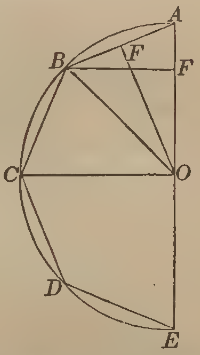
that is, *ED* multiplied into the circumference of the circle described with the radius *NM*, is equal to *HI* into the circumference of the circle described with the radius *ON*. But the former is equal to the surface described by the line *ED* in the revolution of the polygon about the axis *AF*; hence, the latter is equal to the same area; and since the same may be shown for each of the other sides, it is plain that the surface described by the entire perimeter is equal to

$$(FH+HI+IP+PQ+QA) \times \textit{cir'f. } ON = AF \times \textit{cir'f. } ON.$$

Cor. The surface described by any portion of the perimeter, as *EDC*, is equal to the distance between the two perpendiculars let fall from its extremities, on the axis, multiplied by the circumference of the inscribed circle. For, the surface described by *DE* is equal to $HI \times$ circumference *ON*. and the surface described by *DC* is equal to $IP \times$ circumference ON: hence, the surface described by $ED+DC$, is equal to $(HI+IP)\times$ circumference ON, or equal to $HP\times$ circumference ON.

THEOREM XXIII.

The surface of a sphere is equal to the product of its diameter by the circumference of a great circle.



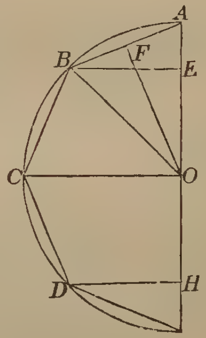
Let $ABCDE$ be a semicircle. Inscribe in it any regular semi-polygon, and from the centre O draw OF perpendicular to one of the sides.

Let the semicircle and the semi-polygon be revolved about the axis AE: the semicircumference $ABCDE$ will describe the surface of a sphere (Def. 26); and the perimeter of the semi-polygon will describe a surface which has for its measure $AE\times$ circumference OF (Th. xxii); and this will be true whatever be the number of sides of the polygon. But if the number of sides of the polygon be indefinitely increased, its perimeter will coincide with the circumference $ABCDE$, the perpendicular OF will become equal to OE, and the surface described by the perimeter of the semi-polygon will then be the same as that described by the semicircumference $ABCDE$. Hence, the surface of the sphere is equal to $AE\times$ circumference OE.

Cor. Since the area of a great circle is equal to the product of its circumference by half the radius, or by one-fourth of the diameter (Bk. IV. Th. xxvii), it follows that the surface of a sphere is equal to four of its great circles.

THEOREM XXIV.

The surface of a zone is equal to its altitude multiplied by the circumference of a great circle.



For, the surface described by any portion of the perimeter of the inscribed polygon, as $BC+CD$ is equal to $EH\times$ circumference OF (Th. xxii. Cor). But when the number of sides of the polygon is indefinitely increased, $BC+CD$, becomes the arc BCD, OF becomes equal to OA, and the surface described by $BC+CD$, becomes the surface of the zone described by the arc BCD: hence, the surface of the zone is equal to $EH\times$ circumference OA.

Sch. 1. When the zone has but one base, as the zone described by the arc $ABCD$, its surface will still be equal to the altitude AE multiplied by the circumference of a great circle.

Sch. 2. Two zones taken in the same sphere, or in equal spheres, are to each other as their altitudes; and any zone is to the surface of the sphere as the altitude of the zone is to the diameter of the sphere.

THEOREM XXV.

The solidity of a sphere is equal to one third of the product of the surface multiplied by the radius.

For, conceive a polyedron to be inscribed in the sphere.

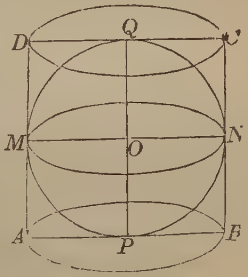
This polyedron may be considered as formed of pyramids, each having for its vertex the centre of the sphere, and for its base one of the faces of the polyedron. Now, the solidity of each pyramid, will be equal to one third of the product of its base by its altitude (Th. xvii).

But if we suppose the faces of the polyedron to be continually diminished, and consequently, the number of the pyramids to be constantly increased, the polyedron will finally become the sphere, and the bases of all the pyramids will become the surface of the sphere. When this takes place, the solidities of the pyramids will still be equal to one third the product of the bases by the common altitude, which will then be equal to the radius of the sphere.

Hence, the solidity of a sphere is equal to one third of the product of the surface by the radius.

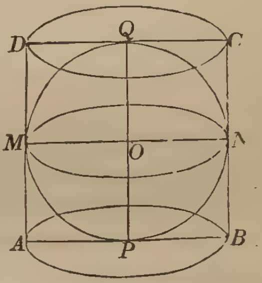
THEOREM XXVI.

The surface of a sphere is equal to the convex surface of the circumscribing cylinder; and the solidity of the sphere is two thirds the solidity of the circumscribing cylinder.



Let *MPNQ* be a great circle of the sphere; *ABCD* the circumscribing square: if the semicircle *PMQ*, and the half square *PADQ*, are at the same time made to revolve about the diameter *PQ*, the semicircle will describe the sphere, while the half square will describe the cylinder circumscribed about that sphere.

The altitude *AD*, of the cylinder, is equal to the diameter



PQ; the base of the cylinder is equal to the great circle, since its diameter *AB* is equal *MN;* hence, the convex surface of the cylinder is equal to the circumference of the great circle multiplied by its diameter (Th. ii). This measure is the same as that of the surface of the sphere (Th. xxiii): hence, the surface of the sphere is equal to the convex surface of the circumscribing cylinder.

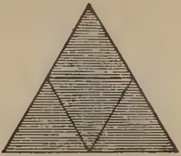
In the next place, since the base of the circumscribing cylinder is equal to a great circle, and its altitude to the diameter, the solidity of the cylinder will be equal to a great circle multiplied by a diameter (Th. xiv. Cor). But the solidity of the sphere is equal to its surface multiplied by a third of its radius; and since the surface is equal to four great circles (Th. xxiii. Cor.), the solidity is equal to four great circles multiplied by a third of the radius; in other words, to one great circle multiplied by four-thirds of the radius, or by two-thirds of the diameter; hence, the sphere is two-thirds of the circumscribing cylinder.

APPENDIX

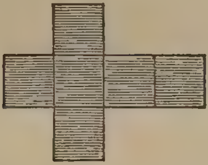
OF THE FIVE REGULAR POLYEDRONS.

A *regular polyedron*, is one whose faces are all equal polygons, and whose polyedral angles are equal. There are five such solids.

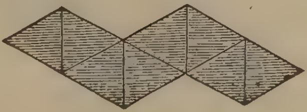
1. The *Tetraedron*, or equilateral pyramid, is a solid bounded by four equal triangles.



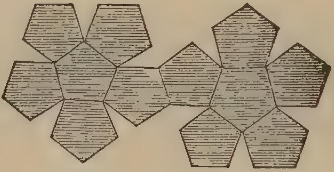
2. The *hexaedron* or *cube*, is a solid, bounded by six equal squares.



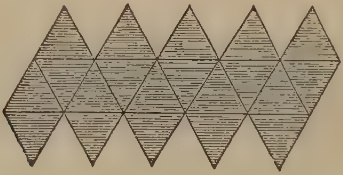
3. The *octaedron*, is a solid, bounded by eight equal equilateral triangles.



4. The *dodecaedron*, is a solid bounded by twelve equal pentagons.



5. The *icosaedron*, is a solid, bounded by twenty equal equilateral triangles.



6. The regular solids may easily be made of pasteboard.

Draw the figures of the regular solids accurately on pasteboard, and then cut through the bounding lines: this will give figures of pasteboard similar to the diagrams. Then, cut the other lines half through the pasteboard, after which, turn up the parts, and glue them together, and you will form the bodies which have been described.

ELEMENTS OF TRIGONOMETRY.

INTRODUCTION.

SECTION I.

OF LOGARITHMS.

1. *The logarithm of a number is the exponent of the power to which it is necessary to raise a fixed number, in order to produce the first number.*

This fixed number is called the *base* of the system, and may be any number except 1: in the common system 10 is assumed as the base.

2. If we form those powers of 10, which are denoted by entire exponents, we shall have

$$10^0=1 \qquad 10^1=10 \quad , \quad 10^3=1000$$

$$10^2=100 \quad , \quad 10^4=10000, \text{ \&c. \&c.}$$

From the above table, it is plain, that 0, 1, 2, 3, 4, &c., are respectively the logarithms of 1, 10, 100, 1000, 10000, &c.; we also see that the logarithm of any number between 1 and 10 is greater than 0 and less than 1: thus

$$\text{Log } 2=0.301030$$

The logarithm of any number greater than 10, and less than 100, is greater than 1 and less than 2: thus

$$\text{Log } 50 = 1.698970$$

The logarithm of any number greater than 100, and less than 1000, is greater than 2 and less than 3: thus

$$\text{Log } 126 = 2.100371, \&c.$$

If the above principles be extended to other numbers, it will appear, that the logarithm of any number, not an exact power of ten, is made up of two parts, *an entire* and a *decimal part.* The *entire part* is called the *characteristic of the logarithm*, and is always one less than the number of places of figures in the given number.

3. The principal use of logarithms, is to abridge numerical computations.

Let M denote any number, and let its logarithm be denoted by m; also let N denote a second number whose logarithm is n; then from the definition we shall have

$$10^m = M \ (1) \qquad 10^n = N \ (2)$$

Multiplying equations (1) and (2), member by member, we have

$$10^{m+n} = M \times N \quad \text{or, } m+n = \log M \times N: \quad \text{hence,}$$

The sum of the logarithms of any two numbers is equal to the logarithm of their product.

Dividing equation (1) by equation (2), member by member, we have

$$10^{m-n} = \frac{M}{N} \quad \text{or,} \quad m-n = \log \frac{M}{N}: \quad \text{hence,}$$

The logarithm of the quotient of two numbers, is equal to the logarithm of the dividend diminished by the logarithm of the divisor.

4. Since the logarithm of 10 is 1, *the logarithm of the product of any number by* 10, *will be greater by* 1 *than the logarithm of that number;* also, *the logarithm of any number divided by* 10, *will be less by* 1 *than the logarithm of that number.*

Similarly, it may be shown that the logarithm of any number multiplied by a hundred, is greater by 2 than the logarithm of that number, and the logarithm of any number divided by 100 is less by 2, than the logarithm of that number, and so on.

EXAMPLES.

log 327	is	2.514548
log 32.7	"	1.514548
log 3.27	"	0.514548
log .327	"	$\bar{1}.514548$
log .0327	"	$\bar{2}.514548$

From the above examples, we see, that in a number composed of an entire and decimal part, we may change the place of the decimal point without changing the decimal part of the logarithm, but *the characteristic is diminished by* 1 *for every place that the decimal point is removed to the left.*

In the logarithm of a decimal, the *characteristic* becomes negative, and is numerically 1 greater than the number of ciphers immediately after the decimal point. The negative sign extends only to the characteristic, and is written over it as in the examples given above.

TABLE OF LOGARITHMS.

5. A table of logarithms, is a table in which are written the logarithms of all numbers between 1 and some given number. The logarithms of all numbers between 1 and 10,000 are given

in the annexed table. Since rules have been given for determining the characteristics of logarithms by simple inspection, it has not been deemed necessary to write them in the table, the decimal part only being given. The characteristic, however, is given for all numbers less than 100.

The left hand column of each page of the table, is the column of numbers, and is designated by the letter N; the logarithms of these numbers are placed opposite them on the same horizontal line. The last column on each page, headed D, shows the difference between the logarithms of two consecutive numbers. This difference is found by subtracting the logarithm under the column headed 4, from the one in the column headed 5 in the same horizontal line, and is nearly a mean of the differences of any two consecutive logarithms on the line.

6. *To find from the table the logarithm of any number.*

If the number is less than 100, look on the first page of the table, in the column of numbers under N, until the number is found: the number opposite is the logarithm sought: Thus

$$\log 9 = 0.954243$$

7. *When the number is greater than* 100 *and less than* 10000.

Find in the column of numbers, the first three figures of the given number. Then pass across the page along a horizontal line until you come into the column under the fourth figure of the given number: at this place, there are four figures of the required logarithm, to which two figures taken from the column marked 0, are to be prefixed.

If the four figures already found stand opposite a row of six figures in the column marked 0, the two left hand figures of the six, are the two to be prefixed; but if they stand opposite a row of only four figures, you ascend the column till you find a row of six figures; the two left hand figures of this row are the two to be prefixed. If you prefix to the decimal part thus found, the characteristic, you will have the logarithm sought: Thus,

$$\log\ 8979 = 3.953228$$
$$\log .08979 = \bar{2}.953228$$

If however in passing back from the four figures found, to the 0 column, any dots be met with, the two figures to be prefixed must be taken from the horizontal line directly below: Thus,

$$\log\ 3098 = 3.491081$$
$$\log 30.98 = 1.491081$$

If the logarithm falls at a place where the dots occur, 0 must be written for each dot, and the two figures to be prefixed are as before taken from the line below: Thus,

$$\log\ 2188 = 3.340047$$
$$\log .2188 = \bar{1}.340047$$

8. *When the number exceeds* 10,000.

The characteristic is determined by the rules already given. To find the decimal part of the logarithm. Place a decimal point after the fourth figure from the left hand, converting the given number into a whole number and decimal. Find the logarithm of the entire part by the rule just given, then take from the right hand column of the page, under D, the number on the same horizontal line with the logarithm, and multiply it by the decimal part; add the product thus obtained to the logarithm already found, and the sum will be the logarithm sought.

If, in multiplying the number taken from the column D, the decimal part of the product exceeds .5 let 1 be added to the entire part; if it is less than .5 the decimal part of the product is neglected.

EXAMPLE.

To find log 672887.

The characteristic is 5.; placing a decimal point after the fourth figure from the left, we have 6728.87. The decimal part of the log 6728 is .827886 and the corresponding number in the column D is 65; then $65 \times .87 = 56.55$, and since the decimal part exceeds .5, we have 57 to be added to 827886, which gives .827943

$$\text{or} \quad \log 672887 = 5.827943$$

Similarly

$$\log .0672887 = \bar{2}.827943$$

The last rule has been deduced under the supposition that the difference of the numbers is proportional to the difference of their logarithms, which is sufficiently exact within the narrow limits considered.

In the above example, 65 is the difference between the logarithm of 672900 and the logarithm of 672800, that is, it is the difference between the logarithms of two numbers which differ by 100.

We have then the proportion 100 : 87 : : 65 . 56.55, the number to be added to the logarithm already found.

9. *To find from the table the number corresponding to a given logarithm.*

Search in the columns of logarithms for the decimal part of the given logarithm: if it cannot be found in the table, take out the number corresponding to the next less logarithm and set it aside. Subtract this less logarithm from the given logarithm, and annex to the remainder as many zeros as may be necessary, and divide this result by the corresponding number taken from the column marked D, continuing the division as long as desirable: annex the quotient to the number set aside. Point off, from the left hand, as many integer figures as there are units in the characteristic of the given logarithm increased by 1; the result is the required number.

If the characteristic is negative, the number will be entirely decimal, and the number of zeros to be placed immediately after the decimal point will be equal to the number of units in the characteristic diminished by 1.

This rule, like its converse, is founded on the supposition that the difference of the logarithms is proportional to the difference of their numbers within narrow limits.

EXAMPLE.

Find the number corresponding to the logarithm 3.233568.

The decimal part of the given logarithm is .233568

The next less logarithm of the table is .233504 and its corresponding number 1712.

Their difference is - - - 64

Tabular difference 253)6400000(25

Hence the number sought 1712.25

The number corresponding to $\bar{3}.233568$ is .00171225

MULTIPLICATION BY LOGARITHMS.

10. When it is required to multiply numbers by means of their logarithms, we first find from the table the logarithms of the numbers to be multiplied; we next add these logarithms together, and their sum is the logarithm of the product of the numbers (Art. 3).

The term *sum* is to be understood in its algebraic sense;

therefore, if any of the logarithms have negative characteristics, the difference between their sum and that of the positive characteristics, is to be taken; the sign of the remainder is that of the greater sum.

EXAMPLES.

1. Multiply 23.14 by 5.062.

log 23.14 = 1.364363
log 5.062 = 0.704322
Product 117.1347 . . . . 2.068685

2. Multiply 3.902, 597.16 and 0.0314728 together.

log 3.902 = 0.591287
log 597.16 = 2.776091
log 0.0314728 = $\bar{2}.497936$
Product 73.3354 . . . . 1.865314

Here the $\bar{2}$ cancels the + 2, and the 1 carried from the decimal part is set down.

3. Multiply 3.586, 2.1046, 0.8372, and 0.0294, together.

log 3.586 = 0.554610
log 2.1046 = 0.323170
log 0.8372 = $\bar{1}.922829$
log 0.0294 = $\bar{2}.468347$
Product 0.1857615 . . $\bar{1}.268956$

In this example the 2, carried from the decimal part, cancels $\bar{2}$, and there remains $\bar{1}$ to be set down.

DIVISION OF NUMBERS BY LOGARITHMS.

11. When it is required to divide numbers by means of their logarithms, we have only to recollect, that the subtraction of

logarithms corresponds to the division of their numbers (Art. 3). Hence, if we find the logarithm of the dividend, and from it subtract the logarithm of the divisor, the remainder will be the logarithm of the quotient.

This additional caution may be added. The difference of the logarithms, as here used, means the *algebraic difference;* so that, if the logarithm of the divisor have a negative characteristic its sign must be changed to positive, after diminishing it by the unit, if any, carried in the subtraction from the decimal part of the logarithm. Or, if the characteristic of the logarithm of the dividend is negative, it must be treated as a negative number.

EXAMPLES.

1. To divide 24163 by 4567.

log 24163 = 4.383151
log 4567 = 3.659631
Quotient 5.29078 . . . . . 0.723520

2. To divide 0.06314 by .007241

log 0.06314 = $\bar{2}.800305$
log 0.007241 = $\bar{3}.859799$
Quotient . . 8.7198 . . . . . 0.940506

Here, 1 carried from the decimal part to the $\bar{3}$ changes it to $\bar{2}$, which being taken from $\bar{2}$, leaves 0 for the characteristic.

3. To divide 37.149 by 523.76

log 37.149 = 1.569947
log 523.76 = 2.719133
Quotient . . 0.0709274 . $\bar{2}.850814$

4. To divide 0.7438 by 12.9476

$$\log\ 0.7438 = \bar{1}.871456$$
$$\log\ 12.9476 = 1.112189$$

Quotient . . 0.057447 . . $\bar{2}.759267$

Here, the 1 taken from $\bar{1}$, gives $\bar{2}$ for a result, as set down.

ARITHMETICAL COMPLEMENT.

12. The *Arithmetical complement* of a logarithm is the number which remains after subtracting the logarithm from 10.

Thus, . . $1-9.274687 = 0.725313$

Hence, 0.725313 is the arithmetical complement of 9.274687.

13. We will now show that, *the difference between two logarithms is truly found, by adding to the first logarithm the arithmetical complement of the logarithm to be subtracted, and then diminishing the sum by* 10.

Let $a =$ the first logarithm

$b =$ the logarithm to be subtracted

and $c = 10-b =$ the arithmetical complement of b.

Now the difference between the two logarithms will be expressed by $a-b$.

But, from the equation $c = 10-b$, we have

$$c-10 = -b$$

hence, if we place for $-b$ its value, we shall have

$$a-b = a+c-10$$

which agrees with the enunciation.

When we wish the arithmetical complement of a logarithm, we may write it directly from the table, *by subtracting the left* *hand figure from* 9, *then proceeding to the right, subtract each figure from* 9 *till we reach the last significant figure, which must be taken from* 10: *this will be the same as taking the logarithm from* 10.

EXAMPLES.

1. From 3.274107 take 2.104729.

	By common method.			*By arith. comp.*
	3.274107			3.274107
	2.104729	its ar. comp.		7.895271
Diff.	1.169378		Sum	1.169378 after subtracting 10.

Hence, to perform division by means of the arithmetical complement we have the following

RULE.

To the logarithm of the dividend add the arithmetical complement of the logarithm of the divisor: the sum after subtracting 10, *will be the logarithm of the quotient.*

EXAMPLES.

1. Divide 327.5 by 22.07.

log 327.5	. . .	2.515211
log 22.07	ar. comp.	8.656198
Quotient . . 14.839	. . . .	1.171409

2. Divide 0.7438 by 12.9476.

log 0.7438	. . . . .	$\bar{1}.871456$
log 12.9476	ar. comp.	8.887811
Quotient . . 0.057447	. . .	$\bar{2}.759267$

In this example, the sum of the characteristics is 8, from which, taking 10, the remainder is $\bar{2}$.

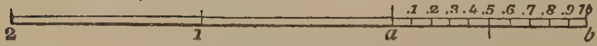
3. Divide 37.149 by 523.76.

log 37.149	. . . . .	1.569947
log 523.76	ar. comp.	7.280867
Quotient . . 0.0709273	. . .	$\bar{2}.850814$

SECTION II.

OF SCALES.

SCALE OF EQUAL PARTS.



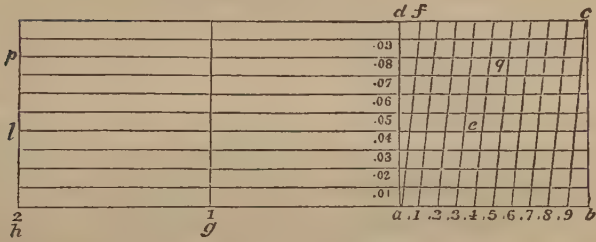
14. A scale of equal parts is formed by dividing a line of a given length into equal portions.

If, for example, the line *ab* of a given length, say one inch, be divided into any number of equal parts, as 10, the scale thus formed, is called a scale of ten parts to the inch. The line *ab*, which is divided, is called the *unit of the scale.* This unit is laid off several times on the left of the divided line, and its extremities marked, 1, 2, 3, &c.

The unit of scales of equal parts, is, in general, either an inch, or an exact part of an inch. If, for example, *ab* the unit of the scale, were half an inch, the scale would be one of 10 parts to half an inch, or of 20 parts to the inch.

If it were required to take from the scale a line equal to two inches and six-tenths, place one foot of the dividers at 2 on the left, and extend the other to .6, which marks the sixth of the small divisions: the dividers will then embrace the required distance.

DIAGONAL SCALE OF EQUAL PARTS.



15. This scale is thus constructed. Take *ab* for the unit of the scale, which may be one inch, $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{3}{4}$ of an inch, in length. On *ab* describe the square *abcd*. Divide the sides *ab* and *dc* each into ten equal parts. Draw *af* and the other nine parallels as in the figure.

Produce *ba* to the left, and lay off the unit of the scale any convenient number of times, and mark the points 1, 2, 3, &c. Then, divide the line *ad* into ten equal parts, and through the points of division draw parallels to *ab* as in the figure.

Now, the small divisions of the line *ab* are each one-tenth (.1) of *ab*; they are therefore .1 of *ad*, or .1 of *ag* or *gh*.

If we consider the triangle *adf*, we see that the base *df* is one-tenth of *ad*, the unit of the scale. Since the distance from *a* to the first horizontal line above *ab*, is one-tenth of the distance *ad*, it follows that the distance measured on that line between *ad* and *af* is one-tenth of *df:* but since one-tenth of a tenth is a hundredth, it follows that this distance is one-hundredth (.01) of the unit of the scale. A like distance measured on the second line will be two-hundredths (.02) of the unit of the scale; on the third, .03; on the fourth, .04, &c.

If it were required to take, in the dividers, the unit of the scale, and any number of tenths, place one foot of the dividers at 1, and extend the other to that figure between *a* and *b* which designates the tenths. If two or more units are required, the dividers must be placed on a point of division further to the left.

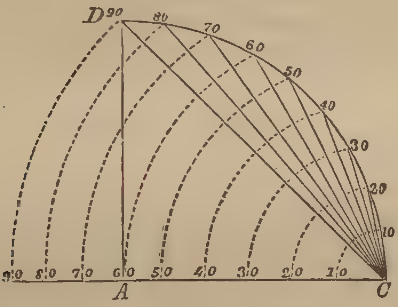
When units, tenths, and hundredths, are required, place one foot of the dividers where the vertical line through the point which designates the units, intersects the line which designates the hundredths: then, extend the dividers to that line between *ad* and *bc* which designates the tenths: the distance so determined will be the one required.

For example, to take off the distance 2.34, we place one foot of the dividers at *l*, and extend the other to *e:* and to take off the distance 2.58, we place one foot of the dividers at *p* and extend the other to *q*.

Remark I. If a line is so long that the whole of it cannot be taken from the scale, it must be divided, and the parts of it taken from the scale in succession.

Remark II. If a line be given upon the paper, its length can be found by taking it in the dividers and applying it to the scale.

SCALE OF CHORDS

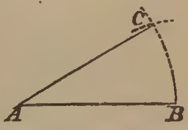


16. If, with any radius, as AC, we describe the quadrant CD, and then divide it into 90 equal parts, each part is called a degree.

Through C, and each point of division, let a chord be drawn, and let the lengths of these chords be accurately laid off on a scale: such a scale is called a *scale of chords.* In the figure, the chords are drawn for every ten degrees.

The scale of chords being once constructed, the radius of the circle from which the chords were obtained, is known; for, the chord marked 60 is always equal to the radius of the circle. A scale of chords is generally laid down on the scales which belong to cases of mathematical instruments, and is marked CHO.

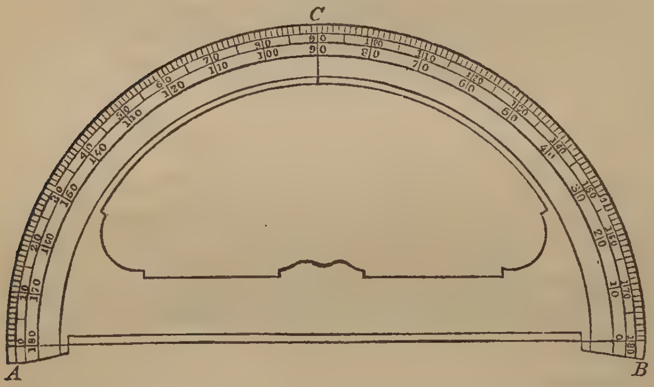
To lay off, at a given point of a line, with the scale of chords, an angle equal to a given angle.



Let AB be the line, and A the given point.

Take from the scale the chord of 60 degrees, and with this radius, and the point A as a centre, describe the arc BC. Then take from the scale the chord of the given angle, say 30 degrees, and with this line as a radius, and B as a centre, describe an arc cutting BC in C. Through A and C draw the line AC, and BAC will be the required angle.

SEMICIRCULAR PROTRACTOR.



17. This instrument is used to lay down, or protract angles. It may also be used to measure angles included between lines already drawn upon paper.

It consists of a brass semicircle ABC divided to half degrees. The degrees are numbered from 0 to 180, both ways; that is, from A to B and from B to A. The divisions, in the figure, are only made to degrees. There is a small notch at the middle of the diameter AB, which indicates the centre of the protractor.

GUNTERS' SCALE.

18. This is a scale of two feet in length, on the faces of which a variety of scales is marked. The face on which the

divisions of inches are made, contains, however, all the scales necessary for laying down lines and angles. These are, the scale of equal parts, the diagonal scale of equal parts, and the scale of chords, all of which have been described.

PLANE TRIGONOMETRY.

DEFINITIONS AND EXPLANATION OF TABLES.

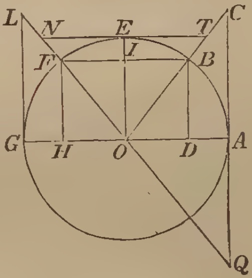
19. In every plane triangle there are six parts: three sides and three angles. These parts are so related to each other, that when one side and any two other parts are given, the remaining parts can be obtained, either by geometrical construction or by trigonometrical computation.

20. *Plane Trigonometry* explains the methods of computing the unknown parts of a plane triangle, when a sufficient number of the six parts is given.

21. For the purpose of trigonometrical calculation, the circumference of the circle is supposed to be divided into 360 equal parts, called degrees; each degree is supposed to be divided into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds.

Degrees, minutes, and seconds, are designated respectively, by the characters ° ′ ″. For example, *ten degrees, eighteen minutes, and fourteen seconds*, would be written 10° 18′ 14″

If two lines be drawn through the centre of the circle, at right angles to each other, they will divide the circumference into four equal parts, of 90° each. Every right angle then, as EOA, is measured by an arc of 90°; every acute angle, as BOA, by an arc less than 90°; and every obtuse angle, as FOA, by an arc greater than 90°.



22. The *complement* of an arc is what remains after subtracting the arc from 90°. Thus, the arc EB is the complement of AB. The sum of an arc and its complement is equal to 90°.

23. The *supplement* of an arc is what remains after subtracting the arc from 180°. Thus, GF is the supplement of the arc AEF. The sum of an arc and its supplement is equal to 180°.

24. The *sine* of an arc is the perpendicular let fall from one extremity of the arc on the diameter which passes through the other extremity. Thus, BD is the sine of the arc AB.

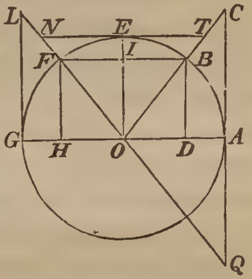
25. The *cosine* of an arc is the part of the diameter intercepted between the foot of the sine and centre. Thus, OD is the cosine of the arc AB.

26. The *tangent* of an arc is the line which touches it at one extremity, and is limited by a line drawn through the other extremity and the centre of the circle. Thus, AC is the tangent of the arc AB.

27. The *secant* of an arc is the line drawn from the centre of the circle through one extremity of the arc, and limited by the tangent passing through the other extremity. Thus, OC is the secant of the arc AB.

28. The four lines, BD, OD, AC, OC, depend for their values on the arc AB and the radius OA; they are thus designated:

sin AB for BD
cos AB for OD
tan AB for AC
sec AB for OC



29. If ABE be equal to a quadrant, or 90°, then EB will be the complement of AB. Let the lines ET and IB be drawn perpendicular to OE. Then,

ET, the tangent of EB, is called the cotangent of AB;
IB, the sine of EB, is equal to the cosine of AB;
OT, the secant of EB, is called the cosecant of AB.

In general, if A is any arc or angle, we have,

$$\cos A = \sin (90° - A)$$
$$\cot A = \tan (90° - A)$$
$$\operatorname{cosec} A = \sec (90° - A)$$

30. If we take an arc $ABEF$, greater than 90°, its sine will be FH; OH will be its cosine; AQ its tangent, and OQ its secant. But FH is the sine of the arc GF, which is the supplement of AF, and OH is its cosine: hence, ***the sine of***

an arc is equal to the sine of its supplement; and the cosine of an arc is equal to the cosine of its supplement.[*]

Furthermore, AQ is the tangent of the arc AF, and OQ is its secant: GL is the tangent, and OL the secant of the supplemental arc GF. But since AQ is equal to GL, and OQ to OL, it follows that, *the tangent of an arc is equal to the tangent of its supplement; and the secant of an arc is equal to the secant of its supplement.*[*]

Let us suppose, that in a circle of a given radius, the lengths of the sine, cosine, tangent, and cotangent, have been calculated for every minute or second of the quadrant, and arranged in a table; such a table is called a table of sines and tangents. If the radius of the circle is 1, the table is called a table of natural sines. A table of natural sines, therefore, shows the values of the sines, cosines, tangents and cotangents of all the arcs of a quadrant, divided to minutes or seconds.

If the sines, cosines, tangents, and secants are known for arcs less than 90°, those for arcs which are greater can be found from them. For if an arc is less than 90°, its supplement will be greater than 90°, and the values of these lines are the same for an arc and its supplement. Thus, if we know the sine of 20°, we also know the sine of its supplement 160°; for the two are equal to each other.

TABLE OF LOGARITHMIC SINES.

31. In this table are arranged the logarithms of the numerical values of the sines, cosines, tangents, and cotangents of all

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[*] These relations are between the numerical *values* of the trigonometrical lines; the algebraic signs, which they have in the different quadrants, are not considered.

the arcs of a quadrant, calculated to a radius of 10,000,000,000. The logarithm of this radius is 10. In the first and last horizontal lines of each page, are written the degrees whose sines, cosines, &c., are expressed on the page. The vertical columns on the left and right, are columns of minutes.

CASE I.

To find, in the table, the logarithmic sine, cosine, tangent, or cotangent of any given arc or angle.

32. If the angle is less than 45°, look for the degrees in the first horizontal line of the different pages: then descend along the column of minutes, on the left of the page, till you reach the number showing the minutes: then pass along the horizontal line till you come into the column designated, sine, cosine, tangent, or cotangent, as the case may be: the number so indicated is the logarithm sought. Thus, on page 37, for 19° 55′ we find,

sine 19° 55′	9.532312
cos 19° 55′	9.973215
tan 19° 55′	9.559097
cot 19° 55′	10.440903

33. If the angle is greater than 45°, search for the degrees along the bottom line of the different pages: then, ascend along the column of minutes on the right hand side of the page, till you reach the number expressing the minutes: then pass along the horizontal line into the column designated tang, cot, sine, or cosine, as the case may be: the number so pointed out is the logarithm required.

34. The column designated sine, at the top of the page, is

designated by cosine at the bottom; the one designated tang, by cotang, and the one designated cotang, by tang.

The angle found by taking the degrees at the top of the page and the minutes from the first vertical column on the left, is the complement of the angle found by taking the degrees at the bottom of the page, and the minutes traced up in the right hand column to the same horizontal line. Therefore, sine, at the top of the page, should correspond with cosine, at the bottom; cosine with sine, tang with cotang, and cotang with tang, as in the tables (Art. 11).

If the angle is greater than 90°, we have only to subtract it from 180°, and take the sine, cosine, tangent or cotangent of the remainder.

The column of the table next to the column of sines, and on the right of it, is designated by the letter *D*. This column is calculated in the following manner.

Opening the table at any page, as 42, the sine of 24° is found to be 9 609313; that of 24° 01′, 9.609597: their difference is 284; this being divided by 60, the number of seconds in a minute, gives 4.73, which is entered in the column *D*.

Now, supposing the increase of the logarithmic sine to be proportional to the increase of the arc, and it is nearly so for 60″, it follows, that 4.73 is the increase of the sine for 1″. Similarly, if the arc were 24° 20′ the increase of the sine for 1″, would be 4.65.

The same remarks are applicable in respect of the column *D*, after the column cosine, and of the column *D*, between the tangents and cotangents. The column *D* between the columns tangents and cotangents, answers to both of these columns.

Now, if it were required to find the logarithmic sine of an arc expressed in degrees, minutes, and seconds, we have only to find the degrees and minutes as before; then, multiply the corresponding tabular difference by the seconds, and add the product to the number first found, for the sine of the given arc.

Thus, if we wish the sine of 40° 26′ 28″.

The sine 40° 26′		9.811952
Tabular difference	2.47	
Number of seconds	28	
Product	69.16 to be added	69.16
Gives for the sine of 40° 26′ 28″		9.812021.

The decimal figures at the right are generally omitted in the final result; but when they exceed five-tenths, the figure on the left of the decimal point is increased by 1; this gives the nearest approximate result.

The tangent of an arc, in which there are seconds, is found in a manner entirely similar. In regard to the cosine and cotangent, it must be remembered, that they increase while the arcs decrease, and decrease as the arcs are increased; consequently, the proportional numbers found for the seconds, must be subtracted, not added.

EXAMPLES.

1. To find the cosine of 3° 40′ 40″

The cosine of 3° 40′		9.999110
Tabular difference	.13	
Number of seconds	40	
Product	5.20 to be subtracted	5.20
Gives for the cosine of 3° 40′ 40″		9.999105

2. Find the tangent of 37° 28′ 31″

Ans. 9.884592.

3. Find the cotangent of 87° 57′ 59″

Ans. 8.550356.

CASE II.

To find the degrees, minutes and seconds, answering to any given logarithmic sine, cosine, tangent or cotangent.

35. Search in the table, and in the proper column, and if the number be found, the degrees will be shown either at the top or bottom of the page, and the minutes in the side columns, either at the left or right.

But, if the number cannot be found in the table, take from the table the degrees and minutes answering to the nearest less logarithm, the logarithm itself, and also the corresponding tabular difference. Subtract the logarithm taken from the table from the given logarithm, annex two ciphers to the remainder, and then divide the remainder by the tabular difference: the quotient will be seconds, and is to be connected with the degrees and minutes before found; to be added for the sine and tangent, and subtracted for the cosine and cotangent.

EXAMPLES.

1. Find the arc answering to the sine 9.880054
Sine 49° 20′, next less in the table 9.879963
Tabular difference . . . 1.81)91.00(50″

Hence, the arc 49° 20′ 50″ corresponds to the given sine 9.880054.

2. Find the arc whose cotangent is . 10.008688
cot 44° 26′, next less in the table . 10.008591
Tabular difference . . . 4.21)97.00(23″

Hence, 44° 26′−23″ = 44° 25′ 37″ is the arc answering to the given cotangent 10.008688.

3. Find the arc answering to tangent 9.979110.

Ans. 43° 37′ 21″.

4. Find the arc answering to cosine 9.944599.

Ans. 28° 19′ 45″.

36. We shall now demonstrate the principal theorems of Plane Trigonometry.

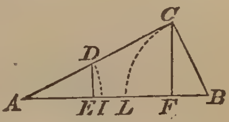
THEOREM I.

The sides of a plane triangle are proportional to the sines of their opposite angles.

Let ABC be a triangle; then will

$$CB : CA :: \sin A : \sin B.$$

For, with A as a centre, and AD equal to the less side BC, as a radius, describe the arc DI: and with B as a centre and the equal radius BC, describe the arc CL: now DE is the sine of the angle A, and CF is the sine of B, to the same radius AD or BC. But by similar triangles,



$$AD : DE :: AC : CF.$$

But AD being equal to BC, we have

$$BC : \sin A :: AC : \sin B, \text{ or}$$

$$BC : AC :: \sin A : \sin B.$$

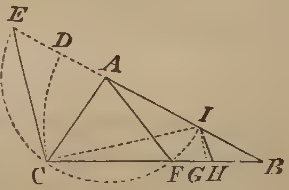
By comparing the sides AB, AC, in a similar manner, we should find, $AB : AC :: \sin C : \sin B$.

THEOREM II.

In any triangle, the sum of the two sides containing either angle, is to their difference, as the tangent of half the sum of the two other angles, to the tangent of half their difference.

Let ACB be a triangle: then will

$AB + AC : AB - AC :: \tan \frac{1}{2}(C + B) : \tan \frac{1}{2}(C - B)$.



With A as a centre, and a radius AC the less of the two given sides, let the semicircle $IFCE$ be described, meeting AB in I, and BA produced, in E. Then, BE will be the sum of the sides, and BI their difference. Draw CI and AF.

Since CAE is an outward angle of the triangle ACB, it is equal to the sum of the inward angles C and B (Bk. I, Th. xvi.) But the angle CIE being at the circumference, is half the angle CAE at the centre (Bk. II, Th. viii. Cor-1); that is, half the sum of the angles C and B, or equal to $\frac{1}{2}(C + B)$.

The angle $AFC = ACB$, is also equal to $ABC + BAF$; therefore, $BAF = ACB - ABC$.

But, $ICF = \frac{1}{2}(BAF) = \frac{1}{2}(ACB - ABC)$, or $\frac{1}{2}(C - B)$.

With I and C as centres, and the common radius IC, let the arcs CD and IG be described, and draw the lines CE and IH perpendicular to IC. The perpendicular CE will pass through E, the extremity of the diameter IE, since the right angle ICE must be inscribed in a semicircle.

But CE is the tangent of $CIE = \frac{1}{2}(C + B)$; and IH is the tangent of $ICB = \frac{1}{2}(C - B)$, to the common radius CI.

But since the lines CE and IH are parallel, the triangles BHI and BCE are similar, and give the proportion,

$$BE : BI :: CE : IH, \text{ or}$$

by placing for BE and BI, CE and IH, their values, we have

$$AB + AC : AB - AC :: \tan \tfrac{1}{2}(C+B) : \tan \tfrac{1}{2}(C - B).$$

THEOREM III.

In any plane triangle, if a line is drawn from the vertical angle perpendicular to the base, dividing it into two segments: then, the whole base, or sum of the segments, is to the sum of the two other sides, as the difference of those sides to the difference of the segments.

Let BAC be a triangle, and AD perpendicular to the base; then will

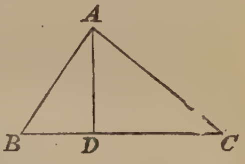
$$BC : CA + AB :: CA - AB : CD - DB$$

For, $\overline{AB}^2 = \overline{BD}^2 + \overline{AD}^2$

(Bk. IV, Th. xii);

and $\overline{AC}^2 = \overline{DC}^2 + \overline{AD}^2$

by subtraction $\overline{AC}^2 - \overline{AB}^2 = \overline{CD}^2 - \overline{BD}^2$.



But since the difference of the squares of two lines is equivalent to the rectangle contained by their sum and difference (Davies' Legendre, Bk. IV, Prop. x,) we have,

$$\overline{AC}^2 - \overline{AB}^2 = (AC + AB) \,.\, (AC - AB)$$

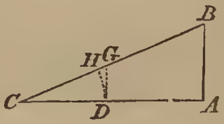
and $\overline{CD}^2 - \overline{DB}^2 = (CD + DB) \,.\, (CD - DB)$

therefore, $(CD + DB) \,.\, (CD - DB) = (AC + AB) \,.\, (AC - AB)$

hence, $CD + DB : AC + AB :: AC - AB : CD - DB$.

THEOREM IV.

In any right-angled plane triangle, radius is to the tangent of either of the acute angles, as the side adjacent to the side opposite.



Let CAB be the proposed triangle, and denote the radius by R: then will

$$R : \tan C :: AC : AB.$$

For, with any radius as CD describe the arc DH, and draw the tangent DG.

From the similar triangles CDG and CAB we have

$$CD : DG :: CA : AB; \text{ hence,}$$

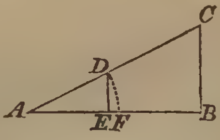
$$R : \tan C :: CA : AB.$$

By describing an arc with B as a centre, we could show in the same manner that,

$$R \cdot \tan B :: AB : AC.$$

THEOREM V.

In every right-angled plane triangle, radius is to the cosine of either of the acute angles, as the hypothenuse to the side adjacent.



Let ABC be a triangle, right-angled at B then will

$$R : \cos A :: AC : AB.$$

For, from the point A as a centre, with any radius as AD, describe the arc DF, which will measure the angle A, and draw DE perpendicular to AB: then will AE be the cosine of A.

The triangles ADE and ACB, being similar, we have

$$AD : AE :: AC : AB: \text{ that is,}$$

$$R : \cos A :: AC : AB.$$

Remark. The relations between the sides and angles of plane triangles, demonstrated in these five theorems, are sufficient to solve all the cases of Plane Trigonometry. Of the six parts which make up a plane triangle, three must be given, and at least one of these a side, before the others can be determined.

If the three angles are given, it is plain, that an indefinite number of similar triangles may be constructed, the angles of which shall be respectively equal to the angles that are given, and therefore, the sides could not be determined.

Assuming, with this restriction, any three parts of a triangle as given, one of the four following cases will always be presented.

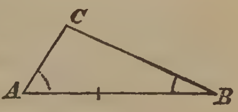
I. When two angles and a side are given.
II. When two sides and an opposite angle are given.
III. When two sides and the included angle are given.
IV. When the three sides are given.

CASE I.

When two angles and a side are given.

Add the given angles together and subtract their sum from 180 degrees. The remaining parts of the triangle can then be found by Theorem I.

EXAMPLES.



1. In a plane triangle ABC, there are given the angle $A = 58° 07'$, the angle $B = 22° 37'$, and the side $AB = 408$ yards. Required the other parts.

GEOMETRICALLY.

Draw an indefinite straight line AB, and from the scale of equal parts lay off AB equal to 408. Then at A lay off an angle equal to 58° 07′, and at B an angle equal to 22° 37′, and draw the lines AC and BC: then will ABC be the triangle required.

The angle C may be measured either with the protractor or the scale of chords (Arts. 16 and 17), and will be found equal to 99° 16′. The sides AC and BC may be measured by referring them to the scale of equal parts (Art. 2). We shall find $AC = 158.9$ and $BC = 351.$ yards.

TRIGONOMETRICALLY BY LOGARITHMS.

To the angle . . $A = 58° 07'$
Add the angle . $B = 22° 37'$
Their sum $= 80° 44'$
taken from . . . 180° 00′
leaves C . . . 99° 16′ which, exceeding 90°
we use its supplement 80° 44′.

To find the side BC.

As sin	C	99° 16′	. ar. comp. .	0.005705
: sin	A	58° 07′	. . . : .	9.928972
: :	AB	408	. . . . .	2.610660
:	BC	351.024	(after rejecting 10)	2.545337

Remark. The logarithm of the fourth term of a proportion is obtained by adding the logarithm of the second term to that of the third, and subtracting from their sum the logarithm of the first term. But to subtract the first term is the same as to add its arithmetical complement and reject 10 from the sum (Art. 13): hence, the arithmetical complement of the first term added to the logarithms of the second and third terms, minus ten, will give the logarithm of the fourth term.

To find side *AC*.

As sin	C	99° 16′	ar. comp. .	0.005705
: sin	B	22° 37′	. . . .	9.584968
: :	AB	408	. . . .	2.610660
:	AC	158.976	. . . .	2.201333

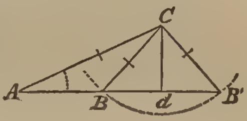
2. In a triangle *ABC*, there are given $A = 38° 25'$, $B = 57° 42'$, and $AB = 400$: required the remaining parts.

Ans. $C = 83° 53'$, $BC = 249.974$, $AC = 340.04$.

CASE II.

When two sides and an opposite angle are given.

In a plane triangle *ABC*, there are given $AC = 216$, $CB = 117$, the angle $A = 22° 37'$, to find the other parts.



GEOMETRICALLY.

Draw an indefinite right line ABB': from any point as A, draw AC making $BAC = 22° 37'$, and make $AC = 216$. With C as a centre, and a radius equal to 117, the other given side, describe the arc $B'B$; draw $B'C$ and BC: then will either of the triangles ABC or $AB'C$, answer all the conditions of the question.

TRIGONOMETRICALLY.

To find the angle B.

As BC	117	ar. comp.	7.931814
: AC	216		2.334454
: : sin A	22° 37′		9.584968
: sin B' 45° 13′ 55″, or ABC 134° 46′ 05″			9.851236

The ambiguity in this, and similar examples, arises in consequence of the first proportion being true for either of the angles ABC, or $AB'C$, which are supplements of each other, and therefore have the same sine (Art. 30). As long as the two triangles exist, the ambiguity will continue. But if the side CB, opposite the given angle, is greater than AC, the arc BB' will cut the line ABB', on the same side of the point A, in but one point, and then there will be only one triangle answering the conditions.

If the side CB is equal to the perpendicular Cd, the arc BB' will be tangent to ABB', and in this case also there will be but one triangle. When CB is less than the perpendicular Cd, the arc BB' will not intersect the base ABB', and in that case, no triangle can be formed, or it will be impossible to fulfil the conditions of the problem.

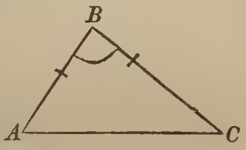
2. Given two sides of a triangle 50 and 40 respectively, and the angle opposite the latter equal to 32°: required the remaining parts of the triangle.

Ans. If the angle opposite the side 50 is acute, it is equal to 41° 28′ 59″; the third angle is then equal to 106° 31′ 01″, and the third side to 72.368. If the angle opposite the side

50 is obtuse, it is equal to 138° 31′ 01″, the third angle to 9° 28′ 59″, and the remaining side to 12.436.

CASE III.

When the two sides and their included angle are given.



Let ABC be a triangle; AB, BC, the given sides, and B the given angle.

Since B is known, we can find the sum of the two other angles: for

$$A + C = 180° - B \text{ and}$$
$$\tfrac{1}{2}(A + C) = \tfrac{1}{2}(180° - B)$$

We next find half the difference of the angles A and C by Theorem ii., viz.

$$BC + BA : BC - BA :: \tan \tfrac{1}{2}(A + C) : \tan \tfrac{1}{2}(A - C):$$

in which we consider BC greater than BA, and therefore A is greater than C; since the greater angle must be opposite the greater side.

Having found half the difference of A and C, by adding it to the half sum, $\frac{1}{2}(A + C)$, we obtain the greater angle, and by subtracting it from half the sum, we obtain the less. That is

$$\tfrac{1}{2}(A + C) + \tfrac{1}{2}(A - C) = A, \text{ and}$$
$$\tfrac{1}{2}(A + C) - \tfrac{1}{2}(A - C) = C.$$

Having found the angles A and C, the third side AC may be found by the proportion.

$$\sin A : \sin B :: BC : AC.$$

EXAMPLES.

1. In the triangle ABC, let $BC = 540$, $AB = 450$, and the included angle $B = 80°$: required the remaining parts.

GEOMETRICALLY.

Draw an indefinite right line BC and from any point, as B, lay off a distance $BC = 540$. At B make the angle $CBA = 80°$: draw BA and make the distance $BA = 450$: draw AC; then will ABC be the required triangle.

TRIGONOMETRICALLY.

$$BC + BA = 540 + 450 = 990; \text{ and } BC - BA = 540 - 450 = 90.$$

$$A + C = 180° - B = 180° - 80° = 100°, \text{ and therefore, } \tfrac{1}{2}(A + C) = \tfrac{1}{2}(100°) = 50°$$

To find $\frac{1}{2}(A - C)$.

As	$BC + BA$	990	ar. comp.	7.004365
:	$BC - BA$	90		1.954243
: :	$\tan \frac{1}{2}(A + C)$	50°		10.076187
:	$\tan \frac{1}{2}(A - C)$	6° 11′		9.034795

Hence, $50° + 6° 11' = 56° 11' = A$; and $50° - 6° 11' = 43° 49' = C$.

To find the third side AC.

As	$\sin C$	43° 49′	ar. comp.	0.159672
:	$\sin B$	80°		9.993351
: :	AB	450		2.653213
:	AC	640.C82		2.806236

2. Given two sides of a plane triangle, 1686 and 960, and their included angle 128° 04′: required the other parts.

Ans. Angles, 33° 34′ 39″; 18° 21′ 21″; side 2400.

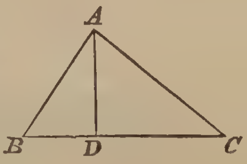
CASE IV.

Having given the three sides of a plane triangle, to find the angles.

Let fall a perpendicular from the angle opposite the greater side, dividing the given triangle into two right-angled triangles: then find the difference of the segments of the base by Theorem iii. Half this difference being added to half the base, gives the greater segment; and, being subtracted from half the base, gives the less segment. Then, since the greater segment belongs to the right-angled triangle having the greatest hypothenuse, we have the sides and right angle of two right-angled triangles, to find the acute angles.

EXAMPLES.

1. The sides of a plane triangle being given; viz. $BC = 40$, $AC = 34$ and $AB = 25$: required the angles.



GEOMETRICALLY.

With the three given lines as sides construct a triangle as in Bk. II. Prob. xi. Then measure the angles of the triangle either with the protractor or scale of chords.

TRIGONOMETRICALLY.

$$\text{As } BC : AC + AB :: AC - AB : CD - BD$$

$$\text{That is, } 40 : 59 :: 9 : \frac{59 \times 9}{40} = 13.275$$

$$\text{Then, } \frac{40 + 13.275}{2} = 26.6375 = CD$$

$$\text{And } \frac{40 - 13.275}{2} = 13.3625 = BD.$$

In the triangle DAC, to find the angle DAC.

As	AC	34	ar. comp.	8.468521
:	DC	26.6375		1.425493
: :	sin D	90°		10.000000
:	sin DAC	51° 34′ 40″		9.894014

In the triangle BAD, to find the angle BAD.

As	AB	25	ar. comp.	8.602060
:	BD	13.3625		1.125887
: :	sin D	90°		10.000000
:	sin BAD	32° 18′ 35″		9.727947

Hence $90° - DAC = 90° - 51° 34' 40'' = 38° 25' 20'' = C$

and $90° - BAD = 90° - 32° 18' 35'' = 57° 41' 25'' = B$

and $BAD + DAC = 51° 34' 40'' + 32° 18' 35'' = 83° 53' 15'' = A$.

2. In a triangle, in which the sides are 4, 5 and 6, what are the angles?

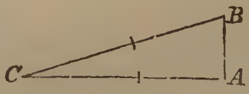
Ans. 41° 24′ 35″; 55° 46′ 16″; and 82° 49′ 09″.

SOLUTION OF RIGHT-ANGLED TRIANGLES.

The unknown parts of a right-angled triangle may be found by either of the four last cases: or, if two of the sides are given, by means of the property that the square of the hypothenuse is equivalent to the sum of the squares of the two other sides. Or the parts may be found by Theorems iv. and v.

EXAMPLES.

1. In a right-angled triangle BAC, there are given the hypothenuse $BC = 250$, and the base $AC = 240$: required the other parts.



To find the angle B.

As	BC	250	ar. comp.	7.602060
:	AC	240		2.380211
: :	sin A	90°		10.000000
:	sin B	73° 44′ 23″		9.982271

But $C = 90° - B = 90° - 73° 44' 23'' = 16° 15' 37''$:

Or C may be found from the proportion.

As	CB	250	ar. comp.	7.602060
:	AC	240		2.380211
: :	R			10.000000
:	cos C	16° 15′ 37″		9.982271

To find side AB by Theorem iv.

As	R		ar. comp.	0.000000
:	tan C	16° 15′ 37″		9.464889
: :	AC	240		2.380211
:	AB	70.0003		1.845100

2. In a right-angled triangle BAC, there are given $AC = 384$, and $B = 53° 08'$: required the remaining parts.

Ans. $AB = 287.96$; $BC = 479.979$; $C = 36° 52'$.

DEFINITIONS.

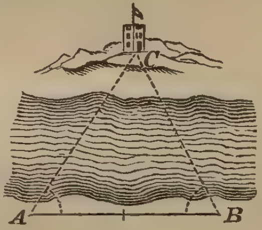
1. A *horizontal angle* is one whose sides are horizontal; its plane is also horizontal.

2. An angle of *elevation* or *depression*, has one horizontal side, and the other oblique, but lying directly above or below the first.

APPLICATION TO HEIGHTS AND DISTANCES.

PROBLEM I.

To determine the horizontal distance to a point which is inaccessible by reason of an intervening river.



Let C be the point. Measure along the bank of the river a horizontal base line AB, and select the stations A and B, in such a manner that each can be seen from the other, and the point C from both of them. Then measure the horizontal angles CAB and CBA, with an instrument adapted to that purpose.

Let us suppose that we have found $AB = 600$ yards, $CAB = 57° 35'$ and $CBA = 64° 51'$.

$$\text{The angle } C = 180° - (A + B) = 57° 34'.$$

To find the distance BC.

As	sin C	57° 34′	ar. comp.	0.073649
:	sin A	57° 35′		9.926431
::	AB	600		2.778151
:	BC	600.11 yards.		2.778231

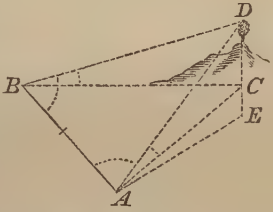
To find the distance AC.

As	sin C	57° 34′	ar. comp.	0.073649
:	sin B	64° 51′		9.956744
::	AB	600		2.778151
:	AC	643.94 yards.		2.808544

PROBLEM II.

To determine the altitude of an inaccessible object above a given horizontal plane.

FIRST METHOD



Suppose D to be the inaccessible object, and BC the horizontal plane from which the altitude is to be estimated: then, if we suppose DC to be a vertical line, it will represent the required distance.

Measure any horizontal base line, as BA; and at the extremities B and A, measure the horizontal angles CBA and CAB. Measure also, the angle of elevation DBC.

Then in the triangle CBA there will be known, two angles and the side AB; the side BC can therefore be determined. Having found BC, we shall have, in the right-angled triangle DBC, the base BC and the angle at the base, to find the perpendicular DC, which measures the altitude of the point D above the horizontal plane BC.

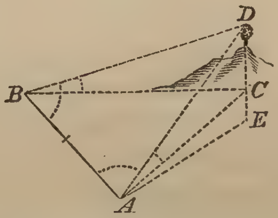
Let us suppose that we have found

$BA = 780$ yards, the horizontal angle $CBA = 41° 24'$, the horizontal angle $CAB = 96° 28'$, and the angle of elevation $DBC = 10° 43'$.

In the triangle BCA, to find the horizontal distance BC.

The angle $BCA = 180° - (41° 24' + 96° 28') = 42° 08' = C$.

As	sin C	42° 08′	ar. comp.	0.173369
:	sin A	96° 28′		9.997228
: :	AB	780		2.892095
:	BC	1155.29		3.062692



In the right-angled triangle DBC, to find DC.

As	R		ar. comp.	0.000000
: tan	DBC	10° 43′		9.277043
: :	BC	1155.29		3.062692
:	DC	218.64		2.339735

REMARK I. It might, at first, appear that the solution which we have given, requires that the points B and A should be in the same horizontal plane; but it is entirely independent of such a supposition.

For, the horizontal distance, which is represented by BA, is the same, whether the station A is on the same level with B, above it, or below it. The horizontal angles CAB and CBA are also the same, so long as the point C is in the vertical line DC. Therefore, if the horizontal line through A should cut the vertical line DC, at any point as E, above or below C, AB would still be the horizontal distance between B and A, and AE which is equal to AC, would be the horizontal distance between A and C.

If at A, we measure the angle of elevation of the point D, we shall know in the right-angled triangle DAE, the base AE, and the angle at the base; from which the perpendicular DE can be determined.

Let us suppose that we had measured the angle of elevation DAE, and found it equal to 20° 15′.

First: In the triangle BAC, to find AC or its equal AE.

As sin C	42° 08′	ar. comp.	0.173369
: sin B	41° 24′		9.820406
: : AB	780		2.892095
: AC	768.9		2.885870

In the right-angled triangle DAE, to find DE.

As R		ar. comp.	0.000000
: tan A	20° 15′		9.566932
: : AE	768.9		2.885870
: DE	283.66		2.452802

Now, since DC is less than DE, it follows that the station B is above the station A. That is,

$$DE - DC = 283.66 - 218.64 = 65.02 = EC,$$

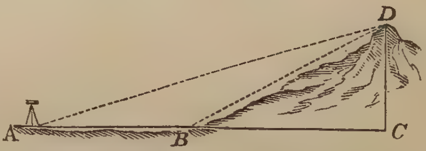
which expresses the vertical distance that the station B is above the station A.

Remark II. It should be remembered, that the vertical distance which is obtained by the calculation, is estimated from a horizontal line passing through the eye at the time of observation. Hence, the height of the instrument is to be added, in order to obtain the true result.

SECOND METHOD.

When the nature of the ground will admit of it, measure a base line AB in the direction of the object D. Then measure with the instrument the angles of elevation at A and B.

Then, since the outward angle DBC is equal to the sum of the angles A and ADB, it follows, that the angle ADB is equal to the difference of the angles of elevation at A and B. Hence, we can find all the parts of the triangle ADB. Having found DB, and knowing the angle DBC, we can find the altitude DC.



This method supposes that the stations A and B are on the same horizontal plane; and therefore can only be used when the line AB is nearly horizontal.

Let us suppose that we have measured the base line, and the two angles of elevation, and

$$\text{found} \begin{cases} AB = 975 \text{ yards}, \\ A = 15^\circ\ 36', \\ DBC = 27^\circ\ 29'; \end{cases}$$

required the altitude DC.

First: $ADB = DBC - A = 27^\circ\ 29' - 15^\circ\ 36' = 11^\circ\ 53'$.

In the triangle ADB, to find BD.

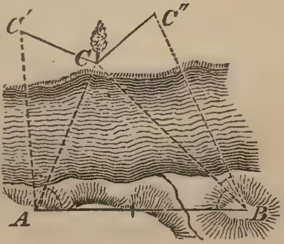
As	sin D	11° 53′	ar. comp.	0.686302
:	sin A	15° 36′		9.429623
: :	AB	975		2.989005
:	DB	1273.3		3.104930

In the triangle DBC, to find DC.

As	R		ar. comp.	0.000000
:	sin B	27° 29′		9.664163
: :	DB	1273.3		3.104930
:	DC	587.61		2.769093

PROBLEM III.

To determine the perpendicular distance of an object below a given horizontal plane.



Suppose C to be directly over the given object, and A the point through which the horizontal plane is supposed to pass.

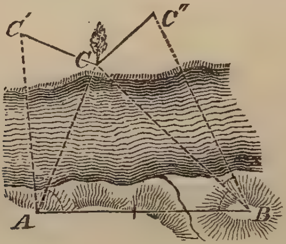
Measure a horizontal base line AB, and at the stations A and B conceive the two horizontal lines AC, BC, to be drawn. The oblique lines from A and B to the object will be the hypothenuses of two right-angled triangles, of which AC, BC, are the bases. The perpendiculars of these triangles will be the distances from the horizontal lines AC, BC, to the object. If we turn the triangles about their bases AC, BC, until they become horizontal, the object, in the first case, will fall at C', and in the second at C''.

Measure the horizontal angles CAB, CBA, and also the angles of depression $C'AC$, $C''BC$.

Let us suppose that we have

$$\text{found} \begin{cases} AB = 672 \text{ yards} \\ BAC = 72° \, 29' \\ ABC = 39° \, 20' \\ C'AC = 27° \, 49' \\ C''BC = 19° \, 10' \end{cases}$$

First: In the triangle ABC, the horizontal angle $ACB = 180° - (A + B) = 180° - 111° \, 49' = 68° \, 11'$.



To find the horizontal distance *AC*.

As	sin *C*	68° 11′	ar. comp.	0.032275
:	sin *B*	39° 20′		9.801973
: :	*AB*	672		2.827369
:	*AC*	458.79		2.661617

To find the horizontal distance *BC*.

As	sin *C*	68° 11′	ar. comp.	0.032275
:	sin *A*	72° 29′		9.979380
: :	*AB*	672		2.827369
:	*BC*	690.28		2.839024

In the triangle *CAC′*, to find *CC′*.

As	*R*		ar. comp.	0.000000
:	tan *C′AC*	27° 49′		9.722315
: :	*AC*	458.79		2.661617
:	*CC′*	242.06		2.383932

In the triangle *CBC″*, to find *CC″*

As	*R*		ar. comp.	0.000000
:	tan *C″BC*	19° 10′		9.541061
: :	*BC*	690.28		2.839024
:	*CC″*	239.93		2.380085

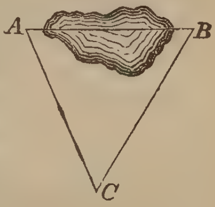
Hence also, $CC' - CC'' = 242.06 - 239.93 = 2.13$ yards, which is the height of the station A above station B.

PROBLEMS.

1. Wanting to know the distance between two inaccessible objects, which lie in a direct line from the bottom of a tower of 120 feet in height, the angles of depression are measured, and are found to be, of the nearer 57°, of the more remote 25° 30′: required the distance between them.

Ans. 173.656 feet.

2. In order to find the distance between two trees A and B, which could not be directly measured because of a pool which occupied the intermediate space, the distances of a third point C from each of them were measured, and also the included angle ACB: it was found that



$$CB = 672 \text{ yards}$$
$$CA = 588 \text{ yards}$$
$$ACB = 55^\circ\ 40';$$

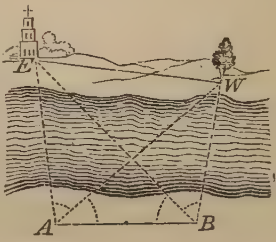
required the distance AB.

Ans. 592.967 yards.

3. Being on a horizontal plane, and wanting to ascertain the height of a tower, standing on the top of an inaccessible hill, there were measured, the angle of elevation of the top of the hill 40°, and of the top of the tower 51°; then measuring in a direct line 180 feet farther from the hill, the angle of elevation of the top of the tower was 33° 45′; required the height of the tower.

Ans. 83.998 feet.

4. Wanting to know the horizontal distance between two inaccessible objects E and W, the following measurements were made,

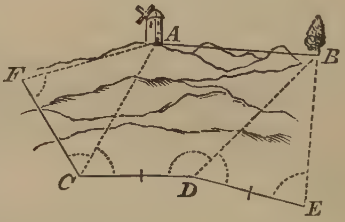


$$\text{viz.}\left\{\begin{aligned} AB &= 536 \text{ yards} \\ BAW &= 40^\circ\ 16' \\ WAE &= 57^\circ\ 40' \\ ABE &= 42^\circ\ 22' \\ EBW &= 71^\circ\ 07' \end{aligned}\right.$$

required the distance EW.

Ans. 939.527 yards.

5. Wanting to know the horizontal distance between two inaccessible objects A and B, and not finding any station from which both of them could be seen, two points C and D, were chosen, at a distance from each other, equal to 200 yards; from the former of these points A could be seen, and from the latter B, and at each of the points C and D a staff was set up. From C a distance CF was measured, not in the direction DC, equal to 200 yards, and from D a distance DE equal to 200 yards, and the following angles taken,



$$\text{viz.}\left\{\begin{aligned} AFC &= 83^\circ\ 00' & BDE &= 54^\circ\ 30' \\ ACD &= 53^\circ\ 30' & BDC &= 156^\circ\ 25' \\ ACF &= 54^\circ\ 31' & BED &= 88^\circ\ 30' \end{aligned}\right.$$

Ans. $AB = 345.467$ yards.

APPLICATIONS

OF

GEOMETRY.

MENSURATION OF SURFACES.

DEFINITIONS.

1. The area of any figure has already been defined to be the measure of its surface (Bk. IV. Def. 7). This measure is merely the number of squares which the figure contains.

A square whose side is one inch, one foot, or one yard, &c., is called the *measuring unit;* and the area or contents of a figure is expressed by the number of such squares which the figure contains.

2. In the questions involving decimals, the decimals are generally carried to four places, and then taken to the nearest figure. That is, if the fifth decimal figure is 5, or greater than 5, the fourth figure is increased by one.

3. Surveyors, in measuring land, generally use a chain called Gunter's chain. This chain is four rods, or 66 feet in length, and is divided into 100 links.

4. An *acre* is a surface equal in extent to 10 square chains; that is, equal to a rectangle of which one side is ten chains and the other side one chain.

One quarter of an acre, is called a *rood.*

Since the chain is 4 rods in length, 1 square chain contains 16 square rods; and therefore, an acre, which is 10 square chains, contains 160 square rods, and a rood contains 40 square rods. The square rods are called perches.

5. Land is generally computed in acres, roods, and perches which are respectively designated by the letters A, R, P.

When the linear dimensions of a survey are chains or links the area will be expressed in square chains or square links, and it is necessary to form a rule for reducing this area to acres, roods, and perches. For this purpose, let us form the following

TABLE.

1 square chain $=100\times100=10000$ square links.
1 acre $=10$ square chains $=100000$ square links.
1 acre $=4$ roods $=160$ perches.
1 square mile $=6400$ square chains $=640$ acres.

6. Now, when the linear dimensions are links, the area will be expressed in square links, and may be reduced to acres by dividing by 100000, the number of square links in an acre: that is, by pointing off five decimal places from the right hand.

If the decimal part be then multiplied by 4, and five places of decimals pointed off from the right hand, the figures to the left hand will express the roods.

If the decimal part of this result be now multiplied by 40, and five places for decimals pointed off, as before, the figures to the left will express the perches.

If one of the dimensions be in links, and the other in chains, the chains may be reduced to links by annexing two ciphers or, the multiplication may be made without annexing the ciphers, and the product reduced to acres and decimals of an acre, by pointing off three decimal places at the right hand.

When both dimensions are in chains, the product is reduced to acres by dividing by 10, or pointing off one decimal place.

From which we conclude: that,

I. *If links be multiplied by links, the product is reduced to acres by pointing off five decimal places from the right hand.*

II. *If chains be multiplied by links, the product is reduced to acres by pointing off three decimal places from the right hand.*

III. *If chains be multiplied by chains, the product is reduced to acres by pointing off one decimal place from the right hand.*

7. Since there are 16.5 feet in a rod, a square rod is equal to $16.5 \times 16.5 = 272.25$ square feet.

If the last number be multiplied by 160, we shall have

$272.25 \times 160 = 43560$ the square feet in an acre.

Since there are 9 square feet in a square yard, if the last number be divided by 9, we obtain

$4840 =$ the number of square yards in an acre.

PROBLEM I.

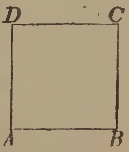
To find the area of a square, a rectangle, a rhombus, or a parallelogram.

RULE.

Multiply the base by the perpendicular height and the product will be the area (Bk. IV. Th. viii).

EXAMPLES.

1 Required the area of the square *ABCD*, each of whose sides is 36 feet



We multiply two sides of the square together, and the product is the area in square feet.

Operation.

$36 \times 36 = 1296$ *sq. ft.*

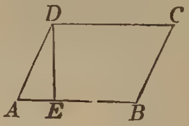
2. How many acres, roods, and perches, in a square whose side is 35.25 chains? *Ans.* 124 *A.* 1 *R.* 1 *P.*

3. What is the area of a square whose side is 8 feet 4 inches? *Ans.* 69 *ft.* 5′ 4″.

4. What is the contents of a square field whose side is 46 rods? *Ans.* 13 *A.* 0 *R.* 36 *P.*

5. What is the area of a square whose side is 4769 yards? *Ans.* 22743361 *sq. yds*

6. What is the area of the parallelogram *ABCD*, of which the base *AB* is 64 feet, and altitude *DE*, 36 feet?



We multiply the base 64, by the perpendicular height 36, and the product is the required area.

Operation.

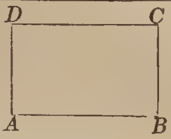
$64 \times 36 = 2304$ *sq. ft.*

7. What is the area of a parallelogram whose base is 12.25 yards, and altitude 8.5? *Ans.* 104,125 *sq. yds.*

8. What is the area of a parallelogram whose base is 8.75 chains, and altitude 6 chains? *Ans.* 5 *A.* 1 *R.* 0 *P.*

9. What is the area of a parallelogram whose base is 7 feet 9 inches, and altitude 3 feet 6 inches? *Ans.* 27 *sq. ft.* 1′ 6″.

10. To find the area of a rectangle $ABCD$, of which the base $AB=45$ yards, and the altitude $AD=15$ yards.



Here we simply multiply the base by the altitude, and the product is the area.

Operation

$45 \times 15 = 675$ *sq. yds.*

11. What is the area of a rectangle whose base is 14 feet 6 inches, and breadth 4 feet 9 inches?

Ans. 68 *sq. ft.* 10′ 6″.

12. Find the area of a rectangular board whose length is 112 feet, and breadth 9 inches. *Ans.* 84 *sq. ft.*

13. Required the area of a rhombus whose base is 10.51 and breadth 4.28 chains. *Ans.* 4 *A.* 1 *R.* 39.7 *P*+.

14. Required the area of a rectangle whose base is 12 feet 6 inches, and altitude 9 feet 3 inches.

Ans. 115 *sq. ft.* 7′ 6″

PROBLEM II.

To find the area of a triangle, when the base and altitude are known.

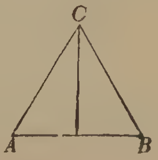
RULE.

I. *Multiply the base by the altitude, and half the product will be the area.*

II. *Multiply the base by half the altitude and the product will be the area* (Bk. IV. Th. ix).

EXAMPLES.

1. Required the area of the triangle ABC, whose base AB is 10,75 feet, and altitude 7,25 feet.



We first multiply the base by the altitude, and then divide the product by 2.

Operation.

$10,75 \times 7,25 = 77,9375$

and

$77,9375 \div 2 = 38,96875$

$=$area.

2. What is the area of a triangle whose base is 18 feet 4 inches, and altitude 11 feet 10 inches?
Ans. 108 *sq. ft.* 5′ 8″.

3. What is the area of a triangle whose base is 12.25 chains, and altitude 8.5 chains? *Ans.* 5 *A.* 0 *R.* 33 *P.*

4. What is the area of a triangle whose base is 20 feet, and altitude 10.25 feet. *Ans.* 102.5 *sq. ft.*

5. Find the area of a triangle whose base is 625 and altitude 520 feet. *Ans.* 162500 *sq. ft.*

6. Find the number of square yards in a triangle whose base is 40 and altitude 30 feet. *Ans.* 66$\frac{2}{3}$ *sq. yds.*

7. What is the area of a triangle whose base is 72.7 yards, and altitude 36.5 yards? *Ans.* 1326,775 *sq. yds*

PROBLEM III.

To find the area of a triangle when the three sides are known.

RULE,

I. *Add the three sides together and take half their sum.*

II. *From this half sum take each side separately.*

III. *Multiply together the half sum and each of the three remainders, and then extract the square root of the product, which will be the required area.*

EXAMPLES.

1. Find the area of a triangle whose sides are 20, 30, and 40 rods.

20	45	45	45
30	20	30	40
40	25 *1st rem.*	15 *2d rem.*	5 *3d rem.*
2)90			
45 half sum,			

Then, to obtain the product, we have

$$45 \times 25 \times 15 \times 5 = 84375;$$

from which we find

$$\text{area} = \sqrt{84375} = 290{,}4737 \text{ perches.}$$

2. How many square yards of plastering are there in a triangle, whose sides are 30, 40, and 50 feet? *Ans.* $66\frac{2}{3}$.

3. The sides of a triangular field are 49 chains, 50.25 chains, and 25.69: what is its area?

Ans. 61 *A.* 1 *R.* 39,68 *P.*

4. What is the area of an isosceles triangle, whose base is 20, and each of the equal sides 15? *Ans.* 111 803.

5. How many acres are there in a triangle whose three sides are 380, 420 and 765 yards. *Ans.* 9 *A.* 0 *R.* 38 *P.*

6. How many square yards in a triangle whose sides are 13, 14, and 15 feet. *Ans.* $9\frac{1}{3}$.

7 What is the area of an equilateral triangle whose side is 25 feet? *Ans.* 270.6329 *sq. ft.*

8. What is the area of a triangle whose sides are 24, 36, and 48 yards? *Ans* 418.282 *sq. yds.*

PROBLEM IV.

To find the hypothenuse of a right angled triangle when the base and perpendicular are known.

RULE.

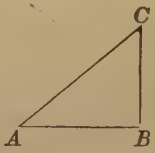
I. *Square each of the sides separately.*

II. *Add the squares together.*

III. *Extract the square root of the sum, which will be the hypothenuse of the triangle* (Bk. IV. Th. xii).

EXAMPLES.

1. In the right angled triangle ABC, we have, $AB=30$ feet, $BC=40$ feet, to find AC.



We first square each side, and then take the sum, of which we extract the square root, which gives

$$AC=\sqrt{2500}=50 \text{ feet.}$$

Operation.

$$\overline{30}^2 = 900$$
$$\overline{40}^2 = 1600$$
$$\text{sum} = 2500$$

2. The wall of a building, on the brink of a river, is 120 feet high, and the breadth of the river 70 yards: what is the length of a line which would reach from the top of the wall to the opposite edge of the river? *Ans.* 241.86 *ft.*

3. The side roofs of a house of which the eaves are of the same height, form a right angle at the top. Now, the length of the rafters on one side is 10 feet, and on the other 14 feet: what is the breadth of the house? *Ans.* 17.204 *ft.*

4. What would be the width of the house, in the last example, if the rafters on each side were 10 feet?

Ans. 14,142 *ft.*

5. What would be the width, if the rafters on each side were 14 feet? *Ans.* 19.7989 *ft.*

PROBLEM V.

When the hypothenuse and one side of a right angled triangle are known, to find the other side

RULE.

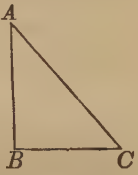
Square the hypothenuse and also the other given side, and take their difference: extract the square root of this difference, and the result will be the required side (Bk. IV. Th. xii. Cor.).

EXAMPLES.

1. In the right angled triangle ABC, there are given

$AC=50$ feet, and $AB=40$ feet,

required the side BC.



We first square the hypothenuse and the other side, after which we take the difference, and then extract the square root, which gives

Operation

$\overline{50}^2=2500$

$\overline{40}^2=1600$

Diff. $=\ \ 900$

$$BC=\sqrt{900}=30 \text{ feet.}$$

2 The height of a precipice on the brink of a river is 103 feet, and a line of 320 feet in length will just reach from the top of it to the opposite bank: required the breadth of the river. *Ans.* 302.9703 *ft.*

3. The hypothenuse of a triangle is 53 yards, and the perpendicular 45 yards: what is the base? *Ans.* 28 *yds.*

4. A ladder 60 feet in length, will reach to a window 40

feet from the ground on one side of the street, and by turning it over to the other side, it will reach a window 50 feet from the ground: required the breadth of the street.

Ans. 77.8875 *ft.*

PROBLEM VI.

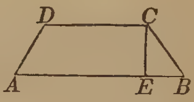
To find the area of a trapezoid.

RULE.

Multiply the sum of the parallel sides by the perpendicular distance between them, and then divide the product by two: the quotient will be the area (Bk. IV. Th. x).

EXAMPLES.

1. Required the area of the trapezoid $ABCD$, having given



$AB=321.51$ feet, $DC=214.24$ feet, and $CE=171.16$ feet

We first find the sum of the sides, and then multiply it by the perpendicular height, after which, we divide the product by 2, for the area.

Operation.

$321.51+214.24=535.75=$ sum of parallel sides.

Then,

$535.75\times171.16=91698.97$

and, $\dfrac{91698.97}{2}=45849.485$ $=$the area.

2. What is the area of a trapezoid, the parallel sides of which, are 12.41 and 8.22 chains and the perpendicular distance between them 5.15 chains?

Ans. 5 *A.* 1 *R.* 9.956 *P.*

3. Required the area of a trapezoid whose parallel sides

are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches.

Ans. 230 *sq. ft.* 5′ 7″.

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and the perpendicular distance between them 10.75 yards. *Ans.* 176.03125 *sq. yds.*

5. What is the area of a trapezoid whose parallel sides are 7.50 chains, and 12.25 chains, and the perpendicular height 15.40 chains? *Ans.* 15 *A.* 0 *R.* 33.2 *P*

PROBLEM VII.

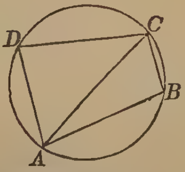
To find the area of a quadrilateral.

RULE.

Measure the four sides of the quadrilateral, and also one of the diagonals: the quadrilateral will thus be divided into two triangles, in both of which all the sides will be known. Then, find the areas of the triangles separately, and their sum will be the area of the quadrilateral.

EXAMPLES.

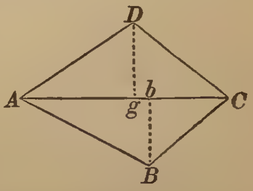
1. Suppose that we have measured the sides and diagonal AC, of the quadrilateral $ABCD$, and found



AB=40.05 chains; CD=29.87 chains,
BC=26.27 chains AD=37.07 chains,
and AC=55 chains:

required the area of the quadrilateral.

Ans. 101 *A.* 1 *R.* 15 *P*



Remark.—Instead of measuring the four sides of the quadrilateral, we may let fall the perpendiculars *Bb*, *Dg*, on the diagonal *AC*. The area of the triangles may then be determined by measuring these perpendiculars and diagonal *AC*. The pendiculars are, *Dg*= 18.95 chains, and *Bb*=17.92 chains.

2. Required the area of a quadrilateral whose diagonal is 60.5, and two perpendiculars 24.5, and 30.1 feet.

Ans. 2197.65 *sq. ft.*

3. What is the area of a quadrilateral whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? *Ans.* 6347 *sq. ft.* 3′.

4. How many square yards of paving in a quadrilateral whose diagonal is 65 feet, and the two perpendiculars 28, and $33\frac{1}{2}$ feet? *Ans.* $222\frac{1}{12}$ *sq. yds.*

5. Required the area of a quadrilateral whose diagonal is 42 feet, and the two perpendiculars 18, and 16 feet.

Ans. 714 *sq. ft.*

6. What is the area of a quadrilateral in which the diagonal is 320.75 chains, and the two perpendiculars 69.73 chains, and 130.27 chains? *Ans.* 3207 *A.* 2 *R.*

PROBLEM VIII.

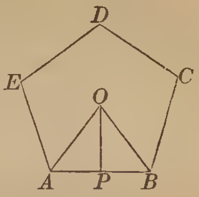
To find the area of a regular polygon.

RULE.

Multiply half the perimeter of the figure by the perpendicular let fall from the centre on one of the sides, and the product will be the area (Bk. IV. Th. xxvi)

EXAMPLES.

1. Required the area of the regular pentagon *ABCDE*, each of whose sides *AB*, *BC*, &c., is 25 feet, and the perpendicular *OP*, 17.2 feet.



We first multiply one side by the number of sides and divide the product by 2: this gives half the perimeter which we multiply by the perpendicular for the area.

Operation.

$\frac{25\times5}{2}=62.5=$half the perimeter. Then,

$62.5\times17.2=1075$ *sq. ft.*=the area.

2. The side of a regular pentagon is 20 yards, and the perpendicular from the centre on one of the sides 13,76382; required the area.

Ans. 688.191 *sq. yds.*

3. The side of a regular hexagon is 14, and the perpendicular from the centre on one of the sides 12.1243556: required the area.

Ans. 509.2229352 *sq. ft.*

4. Required the area of a regular hexagon whose side is 14.6, and perpendicular from the centre 12.64 feet.

Ans. 553.632 *sq ft.*

5. Required the area of a heptagon whose side is 19,38 and perpendicular 20 feet.

Ans. 1356.6 *sq. ft.*

The following table shows the areas of the ten regular polygons when the side of each is equal to 1 : it also shows the length of the radius of the inscribed circle.

Number of sides.	Names.	Areas.	Radius of inscribed circle.
3	Triangle,	0.4330127	0.2886751
4	Square,	1.0000000	0.5000000
5	Pentagon,	1.7204774	0.6881910
6	Hexagon,	2.5980762	0.8660254
7	Heptagon,	3.6339124	1.0382617
8	Octagon,	4,8284271	1.2071068
9	Nonagon,	6.1818242	1.3737387
10	Decagon,	7.6942088	1.5388418
11	Undecagon,	9.3656404	1.2028437
12	Dodecagon,	11,1961524	1.8660254

Now, since the areas of similar polygons are to each other as the squares described on their homologous sides (Bk. IV Th. xx), we have

1^2 : tabular area : : any side squared : area.

Hence, to find the area of a regular polygon, we have the following

RULE.

I *Square the side of the polygon.*

II. *Multiply the square so found, by the tabular area set opposite the polygon of the same number of sides, and the product will be the area.*

EXAMPLES.

1. What is the area of a regular hexagon whose side is 20?

$\overline{20}^2=400$ and tabular area=2,5980762.

Hence,

$2.5980762 \times 400 = 1039.23048 =$ the area.

2. What is the area of a pentagon whose side is 25?
Ans. 1075.298375.

3. What is the area of a heptagon whose side is 30 feet
Ans. 3270.52116

4. What is the area of an octagon whose side is 10 feet?
Ans. 482.84271 *sq. ft*

5. The side of a nonagon is 50: what is its area?
Ans. 15454.5605

6. The side of an undecagon is 20: what is its area?
Ans. 3746.25616.

7. The side of a dodecagon is 40: what is its area?
Ans. 17913.84384.

PROBLEM IX.

To find the area of a long and irregular figure, bounded on one side by a straight line.

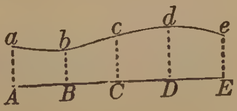
RULE.

I. *Divide the right line or base into any number of equal parts, and measure the breadth of the figure at the points of division, and also at the extremities of the base.*

II. *Add together the intermediate breadths, and half the sum of the extreme ones.*

III. *Multiply this sum by the base line, and divide the product by the number of equal parts of the base.*

EXAMPLES.



1. The breadths of an irregular figure, at five equidistant places, *A*, *B*, *C*, *D*, and *E*, being 8.20 chains, 7.40 chains,

9.20 chains, 10.20 chains, and 8.60 chains, and the whole length 40 chains: required the area.

8.20
8.60
2)16.80
8.40 mean of the extremes.
7.40
9.20
10.20
35.20 the sum.

35.20
40
4)1408.00
352.00 square chains.

Ans. 35 *A.* 32 *P.*

2. The length of an irregular piece of land being 21 chains and the breadths, at six equidistant points, being 4.35 chains 5.15 chains, 3.55 chains, 4.12 chains, 5.02 chains, and 6.10 chains: required the area. *Ans.* 9 *A.* 2 *R.* 30 *P.*

3. The length of an irregular figure is 84 yards, and the breadths at six equidistant places are 17.4; 20.6; 14.2; 16.5; 20.1; and 24.4: what is the area? *Ans.* 1550.64 *sq. yds.*

4. The length of an irregular field is 39 rods, and its breadths at five equidistant places, are 4.8; 5.2; 4.1; 7.3, and 7.2 rods: what is its area? *Ans.* 220.35 *sq. rods.*

5. The length of an irregular field is 50 yards, and its breadths at seven equidistant points, are 5.5; 6.2; 7.3; 6; 7.5; 7; and 8.8 yards: what is its area?

Ans. 342.916 *sq. yds.*

6. The length of an irregular figure being 37.6, and the breadths at nine equidistant places, 0; 4.4; 6.5; 7.6; 5.4; 8; 5.2; 6.5; and 6.1: what is the area? *Ans.* 219.255.

PROBLEM X.

To find the circumference of a circle when the diameter is known.

RULE

Multiply the diameter by 3.1416, *and the product will be the circumference.*

EXAMPLES.

1. What is the circumference of a circle whose diameter is 17?

We simply multiply the number 3.1416 by the diameter, and the product is the circumference.

Operation.

$3.1416 \times 17 = 53.4072$,

which is the circumference.

2. What is the circumference of a circle whose diameter is 40 feet? *Ans.* 125.664 *ft.*

3. What is the circumference of a circle whose diameter is 12 feet? *Ans.* 37.6992 *ft.*

4. What is the circumference of a circle whose diameter is 22 yards? *Ans.* 69.1152 *yds.*

5. What is the circumference of the earth—the mean diameter being about 7921 miles? *Ans.* 24884.6136 *mi.*

PROBLEM XI.

To find the diameter of a circle when the circumference is known.

RULE.

Divide the circumference by the number 3.1416, *and the quotient will be the diameter.*

EXAMPLES.

1. The circumference of a circle is 69.1152 yards: what is the diameter?

We simply divide the circumference by 3.1416, and the quotient 22 is the diameter sought.

Operation.

```
3.1416)69 1152(22
       62832
        62832
        62832
```

2. What is the diameter of a circle whose circumference is 11652.1944 feet? *Ans.* 3709.

3. What is the diameter of a circle whose circumference is 6850? *Ans.* 2180.4176.

4. What is the diameter of a circle whose circumference is 50? *Ans.* 15.915.

5. If the circumference of a circle is 25000.8528, what is the diameter? *Ans.* 7958.

PROBLEM XII.

To find the length of a circular arc, when the number of degrees which it contains, and the radius of the circle are known.

RULE.

Multiply the number of degrees by the decimal .01745, *and the product arising by the radius of the circle.*

EXAMPLES.

1. What is the length of an arc of 30 degrees, in a circle whose radius is 9 feet.

We merely multiply the given decimal by the number of degrees, and by the radius.

Operation.

.01745 × 30 × 9 = 4.7115, which is the length of the arc

Remark.—When the arc contains degrees and minutes, reduce the minutes to the decimals of a degree, which is done by dividing them by 60.

2. What is the length of an arc containing 12° 10′ or $12\frac{1}{6}$°, the diameter of the circle being 20 yards?

Ans. 2.1231

3. What is the length of an arc of 10° 15′ or $10\frac{1}{4}$°, in a circle whose diameter is 68? *Ans.* 6.0813.

PROBLEM XIII.

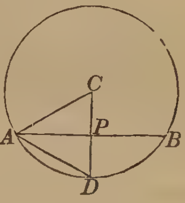
To find the length of the arc of a circle when the chord and radius are given.

RULE.

I. *Find the chord of half the arc.*

II. *From eight times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by* 3, *and the quotient will be the length of the arc, nearly.*

EXAMPLES.



1. The chord $AB=30$ feet, and the radius $AC=20$ feet: what is the length of the arc ADB?

First draw CD perpendicular to the chord AB: it will bisect the chord at P, and the arc of the chord at D. Then $AP=15$ feet. Hence,

$$\overline{AC}^2-\overline{AP}^2=\overline{CP}^2: \text{ that is,}$$

$$400-225=175 \quad \text{and} \quad \sqrt{175}=13.228=CP.$$

Then $CD-CP=20-13.228=6.772=DP.$

Again, $AD=\sqrt{\overline{AP}^2+\overline{PD}^2}=\sqrt{225+45.859984}$

hence, $AD=16.4578=$chord of the half arc.

Then, $\dfrac{16.4578\times8-30}{3}=33.8874=$arc ADB.

2. What is the length of an arc the chord of which is 24 feet, and the radius of the circle 20 feet?

Ans. 25.7309 *ft.*

3. The chord of an arc is 16 and the diameter of the circle 20: what is the length of the arc? *Ans.* 18.5178.

4. The chord of an arc is 50, and the chord of half the arc is 27: what is the length of the arc? *Ans.* $55\frac{1}{3}$.

PROBLEM XIV.

To find the area of a circle when the diameter and circumference are both known.

RULE.

Multiply the circumference by half the radius and the product will be the area (Bk. IV. Th. xxvii).

EXAMPLES.

1. What is the area of a circle whose diameter is 10, and circumference 31.416?

If the diameter be 10, the radius is 5, and half the radius is $2\frac{1}{2}$: hence, the circumference multiplied by $2\frac{1}{2}$ gives the area.

Operation.

$31.416 \times 2\frac{1}{2} = 78.54$;

which is the area.

2. Find the area of a circle whose diameter is 7; and circumference 21.9912 yards. *Ans.* 38.4846 *yds.*

3. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet, and circumference 10.9956. *Ans.* 1.069016.

4. What is the area of a circle whose diameter is 100, and circumference 314.16? *Ans.* 7854

5. What is the area of a circle whose diameter is 1, and circumference 3.1416. *Ans.* 0.7854.

6. What is the area of a circle whose diameter is 40, and circumference 131.9472? *Ans.* 1319.472.

PROBLEM XV.

To find the area of a circle when the diameter only is known.

RULE.

Square the diameter, and then multiply by the decimal .7854.

EXAMPLES.

What is the area of a circle whose diameter is 5?

We square the diameter, which gives us 25, and we then multiply this number and the decimal .7854 together.

Operation.

$$\begin{array}{r} .7854 \\ 5^2 = \quad 25 \\ \hline 39270 \\ 15708 \\ \hline \text{area} = 19.6350 \end{array}$$

2. What is the area of a circle whose diameter is 7? *Ans.* 38.4846.

3. What is the area of a circle whose diameter is 4,5? *Ans.* 15.90435.

4. What is the number of square yards in a circle whose diameter is $1\frac{1}{6}$ yards? *Ans.* 1.069016.

5. What is the area of a circle whose diameter is 8.75 feet? *Ans.* 60.1322 *sq. ft.*

PROBLEM XVI.

To find the area of a circle when the circumference only is known.

RULE.

Multiply the square of the circumference by the decimal .07958, *and the product will be the area very nearly.*

EXAMPLES.

1. What is the area of a circle whose circumference is 3.1416?

We first square the circumference, and then multiply by the decimal .07958.

Operation.

$$\begin{array}{r} \overline{3.1416}^2 = 9,86965056 \\ ,07958 \\ \hline \text{area} = \underline{.7854+} \end{array}$$

2. What is the area of a circle whose circumference is 91?

Ans. 659.00198.

3. Suppose a wheel turns twice in tracking 16½ feet, and that it turns just 200 times in going round a circular bowling-green: what is the area in acres, roods, and perches?

Ans. 4 *A.* 3 *R.* 35.8 *P.*

4. How many square feet are there in a circle whose circumference is 10.9956 yards? *Ans.* 86.5933.

5. How many perches are there in a circle whose circumference is 7 miles? *Ans.* 399300.608.

PROBLEM XVII.

Having given a circle, to find a square which shall have an equal area.

RULE.

I. *The diameter* × .8862 = *side of an equivalent square*

II. *The circumference* × .2821 = *side of an equivalent square.*

EXAMPLES.

1. The diameter of a circle is 100: what is the side of a square of equal area? *Ans.* 88.62.

2. The diameter of a circular fishpond is 20 feet, what would be the side of a square fishpond of an equal area? *Ans.* 17.724 *ft.*

3. A man has a circular meadow of which the diameter is 875 yards, and wishes to exchange it for a square one of equal size: what must be the side of the square? *Ans.* 775.425.

4. The circumference of a circle is 200: what is the side of a square of an equal area? *Ans.* 56.42.

5. The circumference of a round fishpond is 400 yards: what is the side of a square pond of equal area? *Ans.* 112.84.

6. The circumference of a circular bowling-green is 412 yards: what is the side of a square one of equal area? *Ans.* 116.2252 *yds.*

7. The circumference of a circular walk is 625: what is the side of a square containing the same area? *Ans.* 176.3125.

PROBLEM XVIII.

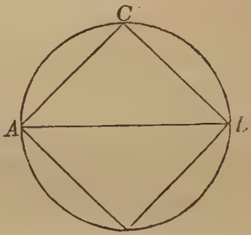
Having given the diameter or circumference of a circle, to find the side of the inscribed square.

RULE.

I. *The diameter* × .7071 = *side of the inscribed square.*

II. *The circumference* × .2251 = *side of the inscribed square.*

EXAMPLES.



1. The diameter AB of a circle is 400: what is the value of AC, the side of the inscribed square?

Here,

$$.7071 \times 400 = 282.8400 = AC.$$

2. The diameter of a circle is 412 feet: what is the side of the inscribed square? *Ans.* 291.3252 *ft.*

3. If the diameter of a circle be 600 what is the side of the inscribed square? *Ans.* 424.26.

4. The circumference of a circle is 312 feet: what is the side of the inscribed square? *Ans.* 70.2312 *ft.*

5. The circumference of a circle is 819 yards: what is the side of the inscribed square? *Ans.* 184.3569 *yds.*

6. The circumference of a circle is 715: what is the side of the inscribed square? *Ans.* 160.9465.

7. The circumference of a circular walk is 625: what is the side of an inscribed square? *Ans.* 140.6875.

PROBLEM XIX.

To find the area of a circular sector.

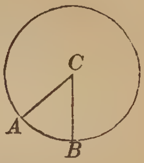
RULE.

I. *Find the length of the arc by Problem* XII.

II. *Multiply the arc by one half the radius, and the product will be the area.*

EXAMPLES.

1. What is the area of the circular sector ACB, the arc AB containing 18°, and the radius CA being equal to 3 feet.



First, $.01745 \times 18 \times 3 = .94230 =$ length AB.
Then, $.94230 \times 1\frac{1}{2} = 1.41345 =$ area

2. What is the area of a sector of a circle in which the radius is 20 and the arc one of 22 degrees?

Ans. 76.7800.

3. Required the area of a sector whose radius is 25 and the arc of 147° 29′. *Ans.* 804.2448.

4. Required the area of a semicircle in which the radius is 13. *Ans.* 265.4143.

5. What is the area of a circular sector when the length of the arc is 650 feet and the radius 325?

Ans. 105625 *sq. ft.*

PROBLEM XX.

To find the area of a segment of a circle.

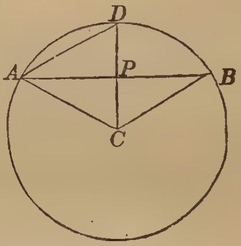
RULE.

I. *Find the area of the sector having the same arc with the segment, by the last Problem.*

II. *Find the area of the triangle formed by the chord of the segment and the two radii through its extremities.*

III. *If the segment is greater than the semicircle, add the two areas together; but if it is less, subtract them, and the result in either case, will be the area required.*

EXAMPLES.



1. What is the area of the segment ADB, the chord $AB=24$ feet and $CA=20$ feet.

First, $CP=\sqrt{\overline{CA}^2-\overline{AP}^2}$

$=\sqrt{400-144}=16$

Then,

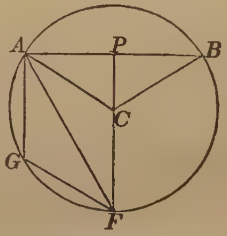
$PD=CD-CP=20-16=4.$

And, $AD=\sqrt{\overline{AP}^2+\overline{PD}^2}=\sqrt{144+16}=12{,}64911:$

then, arc $ADB=\dfrac{12{,}64911\times 8-24}{3}=25{,}7309.$

Arc $ADB=25{,}7309$
half radius $=10$
area sector $ADBC=257{,}3090$
area $CAB=192$
$65{,}309=$area of segment ADB

$AP=12$
$CP=16$
area $CAB=192$



2. Find the area of the segment AFB, knowing the following lines, viz: $AB=20.5$; $FP=17.17$; $AF=20$; $FG=11.5$; and $CA=11.64$.

Arc $AGF=\dfrac{FG\times 8-AF}{3}=\dfrac{11.5\times 8-20}{3}=24:$

and sector $AGFBC=24\times 11.64=279.36:$

but $CP=FP-AC=17.17-11.64=5.53:$

Then, area $ACB=\dfrac{AB\times CP}{2}=\dfrac{20.5\times 5.53}{2}=56.6825.$

Then, area of sector $AFBC=279.36$
do. of triangle $ABC=\ 56.6825$
gives area of segment $AFB=336.0425$

3 What is the area of a segment; the radius of the circle being 10 and the chord of the arc 12 yards?

Ans. 16.324 *sq. yds.*

4. Required the area of the segment of a circle whose chord is 16, and the diameter of the circle 20.

Ans. 44.5903.

5. What is the area of a segment whose arc is a quadrant, the diameter of the circle being 18? *Ans.* 63.6174.

6. The diameter of a circle is 100, and the chord of the segment 60: what is the area of the segment?

Ans. 408, nearly.

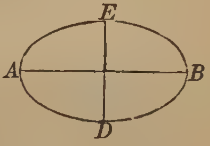
PROBLEM XXI.

To find the area of an ellipse.

Multiply the two axes together, and their product by the decimal ,7854, *and the result will be the required area.*

EXAMPLES.

1. Required the area of an ellipse, whose transverse axis $AB=70$ feet, and the conjugate axis $DE=50$ feet.



$AB\times DE=70\times 50=3500$:

Then, $.7854\times 3500=2748.9=$area.

2. Required the area of an ellipse whose axes are 24 and 18. *Ans.* 339.2928.

3. What is the area of an ellipse whose axes are 80 and 60? *Ans.* 3769.92.

4. What is the area of an ellipse whose axes are 50 and 45? *Ans.* 1767.15.

PROBLEM XXII.

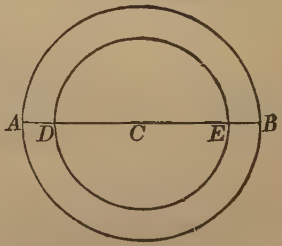
To find the area of a circular ring: that is, the area included between the circumferences of two circles, having a common centre.

RULE.

I. *Square the diameter of each ring, and subtract the square of the less from that of the greater.*

II. *Multiply the difference of the squares by the decimal* ,7854, *and the product will be the area.*

EXAMPLES.



1. In the concentric circles having the common centre *C*, we have

$AB=10$ yds., and $DE=6$ yards: what is the area of the space included between them?

$$\overline{BA}^2=\overline{10}^2=100$$
$$\overline{DE}^2=\overline{6}^2=36$$
$$\text{Difference}=64$$

Then, $63\times.7854=50.2656=$area.

2. What is the area of the ring when the diameters of the circle are 20 and 10? *Ans.* 235.62.

3. If the diameters are 20 and 15, what will be the area included between the circumferences? *Ans.* 137.445.

4. If the diameters are 16 and 10, what will be the area included between the circumferences? *Ans.* 122.5224.

5 Two diameters are 21.75 and 9.5; required the area of the circular ring. *Ans.* 300.6609.

6. If the two diameters are 4 and 6, what is the area of the ring? *Ans.* 15.708

MENSURATION OF SOLIDS.

DEFINITIONS.

The mensuration of solids is divided into two parts.

1st, The mensuration of the surfaces of solids: and

2d, The mensuration of their solidities.

We have already seen that the unit of measure for plane surfaces, is a square whose side is the unit of length (Bk. IV Def. 7).

2. A curve line which is expressed by numbers is also referred to an unit of length, and its numerical value is the number of times which the line contains the unit.

If then, we suppose the linear unit to be reduced to a straight line, and a square constructed on this line, this square will be the unit of measure for curved surfaces.

3. The unit of solidity is a cube, whose edge is the unit in which the linear dimensions of the solid are expressed; and

the face of this cube is the superficial unit in which the surface of the solid is estimated (Bk. VI. Th. xiii. Sch).

4. The following is a table of solid measure.

1 cubic foot $=1728$ cubic inches.
1 cubic yard $=27$ cubic feet.
1 cubic rod $=4492\frac{1}{8}$ cubic feet.
1 ale gallon $=282$ cubic inches.
1 wine gallon $=231$ cubic inches.
1 bushel $=2150{,}42$ cubic inches.

PROBLEM I.

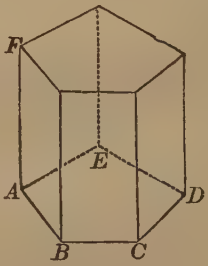
To find the surface of a right prism.

RULE.

Multiply the perimeter of the base by the altitude and the product will be the convex surface: and to this add the area of the bases, when the entire surface is required (Bk. VI. Th. i).

EXAMPLES

1. Find the entire surface of the regular prism whose base is the regular polygon $ABCDE$ and altitude AF, when each side of the base is 20 feet and the altitude AF, 50 feet.



$AB+BC+CD+DE+EA=100$; and $AF=50$: then $(AB+BC+CD+DE+EA)\times AF=$ convex surface which becomes, $100 \times 50 = 5000$ square feet; which is the convex surface. For the area of the end, we have

$$\overline{AB}^2 \times \text{tabular number} = \text{area } ABCDE,$$

that is, $\overline{20}^2 \times$ tabular number, or $400 \times 1.720477 = 688.1908 =$ the area $ABCDE$.

Then, convex surface	=5000	square feet.
lower base	688.1908	square feet.
upper base	688.1908	square feet.
Entire surface	6376.3816	

2. What is the surface of a cube, the length of each side being 20 feet? *Ans.* 2400 *sq. ft.*

3. Find the entire surface of a triangular prism, whose base is an equilateral triangle, having each of its sides equal to 18 inches, and altitude 20 feet. *Ans.* 91.949 *sq. ft.*

4. What is the convex surface of a regular octagonal prism, the side of whose base is 15 and altitude 12 feet?

Ans. 1440 *sq. ft.*

5. What must be paid for lining a rectangular cistern with lead at 2*d* a pound, the thickness of the lead being such as to require 7*lb.* for each square foot of surface; the inner dimensions of the cistern being as follows: viz. the length 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches? *Ans.* £2 3*s*. 10$\frac{5}{8}$*d.*

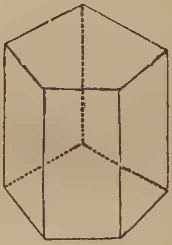
PROBLEM II

To find the solidity of a prism.

RULE.

Multiply the area of the base by the perpendicular height, and the product will be the solidity.

EXAMPLES.



1. What is the solidity of a regular pentagonal prism whose altitude is 20, and each side of the base 15 feet?

To find the area of the base we have by Problem VIII. page 178.

$\overline{15}^2=225$: and $225\times1.7204774=387.107415=$ the area of the base: hence,

$$387.107415\times20=7742.1483=\text{solidity.}$$

2. What is the solid contents of a cube whose side is 24 inches? *Ans.* 13824 *solid in.*

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches? *Ans.* $21\frac{1}{9}$ *solid ft.*

4. How many gallons of water, ale measure, will a cistern contain whose dimensions are the same as in the last example? *Ans.* $129\frac{17}{47}$.

5. Required the solidity of a triangular prism whose altitude is 10 feet, and the three sides of its triangular base 3, 4, and 5 feet. *Ans.* 60 *solid ft.*

6. What is the solidity of a square prism whose height is $5\frac{1}{2}$ feet, and each side of the base $1\frac{1}{3}$ foot? *Ans.* $9\frac{7}{9}$ *solid ft.*

7. What is the solidity of a prism whose base is an equilateral triangle, each side of which is 4 feet, the height of the prism being 10 feet? *Ans.* 69.282 *solid ft.*

8. What is the number of cubic or solid feet in a regular pentagonal prism of which the altitude is 15 feet and each side of the base 3.75 feet? *Ans.* 362.913

PROBLEM III.

To find the surface of a regular pyramid.

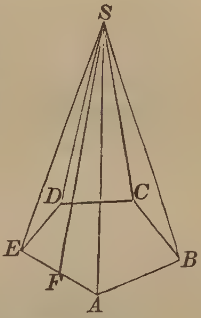
RULE.

Multiply the perimeter of the base by half the slant height, and the product will be the convex surface: to this add the area of the base, if the entire surface is required (Bk. VI. Th vi)

EXAMPLES.

1. In the regular pentagonal pyramid S—$ABCDE$, the slant height SF is equal to 45, and each side of the base is 15 feet: required the convex surface, and also the entire surface.

$15\times5=75=$perimeter of the base,
$75\times22\frac{1}{2}=1687.5$ square feet$=$area of convex surface.



And $\overline{15}^2=225$: then $225\times1.7204774=387.107415=$the area of the base.

Hence, convex surface $=1687.5$
area of the base$=$ 387.107415
Entire surface $=2074.607415$ square feet.

2. What is the convex surface of a regular triangular pyramid, the slant height being 20 feet, and each side of the base 3 feet? *Ans.* 90 *sq. ft*

3. What is the entire surface of a regular pyramid whose slant height is 15 feet, and the base a regular pentagon, of which each side is 25 feet? *Ans.* 2012.798 *sq. ft.*

PROBLEM IV.

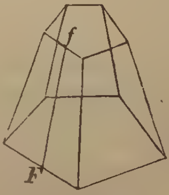
To find the convex surface of the frustum of a regular pyramid.

RULE.

Multiply half the sum of the perimeters of the two bases by the slant height of the frustum, and the product will be the convex surface (Bk. VI. Th. vii).

EXAMPLES.

1. In the frustum of the regular pentagonal pyramid each side of the lower base is 30, and each side of the upper base is 20 feet, and the slant height fF is equal to 15 feet. What is the convex surface of the frustum? *Ans.* 1875 *sq. ft.*



2. How many square feet are there in the convex surface of the frustum of a square pyramid, whose slant height is 10 feet, each side of the lower base 3 feet 4 inches, and each side of the upper base 2 feet 2 inches? *Ans.* 110.

3. What is the convex surface of the frustum of a heptagonal pyramid whose slant height is 55 feet, each side of the lower base 8 feet, and each side of the upper base 4 feet? *Ans.* 2310 *sq. ft.*

PROBLEM V.

To find the solidity of a pyramid.

RULE.

Multiply the area of the base by the altitude and divide the product by 3, the quotient will be the solidity (Bk. VI. Th. xvii).

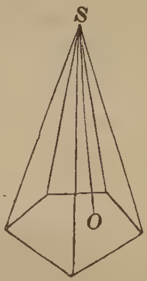
EXAMPLES.

1 What is the solidity of a pyramid the area of whose base is 215 square feet and the altitude $SO=45$ feet?

First, $215 \times 45 = 9675$:

then, $9675 \div 3 = 3225$

which is the solidity expressed in solid feet.



2. Required the solidity of a square pyramid, each side of its base being 30 and its altitude 25. *Ans.* 7500 *solid ft.*

3. How many solid yards are there in a triangular pyramid whose altitude is 90 feet, and each side of its base 3 yards? *Ans.* 38.97117.

4. How many solid feet in a triangular pyramid the altitude of which is 14 feet 6 inches, and the three sides of its base 5, 6 and 7 feet? *Ans.* 71.0352.

5. What is the solidity of a regular pentagonal pyramid, its altitude being 12 feet, and each side of its base 2 feet? *Ans.* 27·5276 *solid ft.*

6 How many solid feet in a regular hexagonal pyramid, whose altitude is 6.4 feet, and each side of the base 6 inches? *Ans.* 1.38564.

7. How many solid feet are contained in a hexagonal pyramid the height of which is 45 feet, and each side of the base 10 feet? *Ans.* 3897.1143.

8. The spire of a church is an octagonal pyramid, each side of the base being 5 feet 10 inches, and its perpendicular height 45 feet. Within is a cavity, or hollow part, each side of the base being 4 feet 11 inches, and its perpendicular height 41 feet: how many yards of stone does the spire contain? *Ans.* 32.197353

PROBLEM VI.

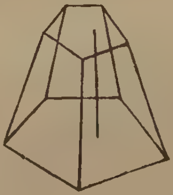
To find the solidity of the frustum of a pyramid.

RULE.

Add together the areas of the two bases of the frustum and a geometrical mean proportional between them; and then multiply the sum by the altitude, and take one-third the product for the solidity.

EXAMPLES.

1. What is the solidity of the frustum of a pentagonal pyramid the area of the lower base being 16 and of the upper base 9 square feet, the altitude being 7 feet?



First, $16 \times 9 = 144$: then, $\sqrt{144} = 12$, the mean

$$\begin{array}{lr}
\text{Then, area of lower base} & =16 \\
\quad\text{area of upper base} & =\ 9 \\
\quad\text{mean of bases} & \underline{=12} \\
 & 37 \\
\quad\text{height} & \underline{7} \\
 & 3)\,\underline{259} \\
\quad\text{solidity} & \underline{=86\tfrac{1}{3}}\ \textit{solid ft.}
\end{array}$$

2. What is the number of solid feet in a piece of timber whose bases are squares, each side of the lower base being 15 inches, and each side of the upper base being 6 inches, the length being 24 feet? *Ans.* 19.5.

3. Required the solidity of a regular pentagonal frustum, whose altitude is 5 feet, each side of the lower base 18 inches, and each side of the upper base 6 inches.

Ans. 9.31925 *solid ft.*

4. What is the contents of a regular hexagonal frustum, whose height is 6 feet, the side of the greater end 18 inches, and of the less end 12 inches? *Ans.* 24.681724 *cubic ft.*

5. How many cubic feet in a square piece of timber, the areas of the two ends being 504 and 372 inches, and its length $31\frac{1}{2}$ feet? *Ans.* 95.447.

6. What is the solidity of a squared piece of timber, its length being 18 feet, each side of the greater base 18 inches, and each side of the smaller 12 inches?

Ans. 28.5 *cubic ft.*

7. What is the solidity of the frustum of a regular hexagonal pyramid, the side of the greater end being 3 feet, that of the less 2 feet, and the height 12 feet?

Ans. 197.453776 *solid ft.*

MEASURES OF THE THREE ROUND BODIES.

PROBLEM I

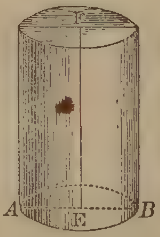
To find the surface of a cylinder.

RULE.

Multiply the circumference of the base by the altitude, and the product will be the convex surface; and to this, add the areas of the two bases, when the entire surface is required (Bk. VI. Th. ii).

EXAMPLES.

1. What is the entire surface of the cylinder in which *AB*, the diameter of the base, is 12 feet, and the altitude *EF* 30 feet?



First, to find the circumference of the base, (Prob. X. page 180): we have $3.1416 \times 12 = 37.6992 =$ circumference of the base.

Then, $37.6992 \times 30 = 1130.9760 =$ convex surface.

Also, $\overline{12}^2 = 144$: and $144 \times .7854 = 113.0976 =$ area of the base.

Then.

convex surface	=1130.9760
lower base	113.0976
upper base	113.0976
Entire area	=1357.1712

2. What is the convex surface of a cylinder, the diameter of whose base is 20, and the altitude 50 feet?

Ans 3141.6 *sq. ft.*

3. Required the entire surface of a cylinder, whose altitude is 20 feet, and the diameter of the base 2 feet.

Ans. 131.9472 *ft.*

4. What is the convex surface of a cylinder, the diameter of whose base is 30 inches, and altitude 5 feet?

Ans. 5654.88 *sq. in.*

5. Required the convex surface of a cylinder, whose altitude is 14 feet, and the circumference of the base 8 feet 4 inches. *Ans.* 116.6666, &c., *sq. ft.*

PROBLEM II.

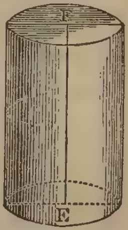
To find the solidity of a cylinder.

RULE.

Multiply the area of the base by the altitude, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a cylinder, the diameter of whose base is 40 feet, and altitude *EF*, 25 feet?



First, to find the area of the base, we have (Prob. xv. page 231).

$\overline{40}^2=1600$: then, $1600\times.7854=1256.64$.
$=$area of the base.

Then, $1256.64\times25=31416$ solid feet, which is the solidity.

2. What is the solidity of a cylinder, the diameter of whose base is 30 feet, and altitude 50 feet?

Ans 35343 *cubic ft.*

3. What is the solidity of a cylinder whose height is 5 feet, and the diameter of the end 2 feet? *Ans.* 15.708 *solid ft.*

4. What is the solidity of a cylinder whose height is 20 feet, and the circumference of the base 20 feet?

Ans. 636.64 *cubic ft.*

5. The circumference of the base of a cylinder is 20 feet, and the altitude 19.318 feet: what is the solidity?

Ans. 614.93 *cubic ft.*

6. What is the solidity of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet?

Ans. 2120.58 *cubic ft.*

7. Required the solidity of a cylinder whose altitude is 20 feet, and the circumference of whose base is 5 feet 6 inches?

Ans. 48.1459 *cubic ft.*

8. What is the solidity of a cylinder, the circumference of whose base is 38 feet, and altitude 25 feet?

Ans. 2872.838 *cubic ft.*

9. What is the solidity of a cylinder, the circumference of whose base is 40 feet, and altitude 30 feet?

10. The diameter of the base of a cylinder is 84 yards, and the altitude 21 feet: how many solid or cubic yards does it contain? *Ans.* 38792.4768.

PROBLEM III.

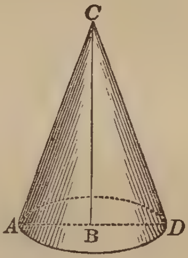
To find the surface of a cone.

RULE.

Multiply the circumference of the base by the slant height, and divide the product by 2; *the quotient will be the convex surface, to which add the area of the base, when the entire surface is required* (Bk. VI. Th. viii).

EXAMPLES.

1. What is the convex surface of the cone whose vertex is C, the diameter AD, of its base being $8\frac{1}{2}$ feet, and the side CA, 50 feet.



First, $3.1416\times 8\frac{1}{2}=26.7036$=circumference of base.

Then $\dfrac{26.7036\times 50}{2}=667.59$=convex surface.

2. Required the entire surface of a cone whose side is 36 and the diameter of its base 18 feet.

Ans. 1272.348 *sq. ft.*

3. The diameter of the base is 3 feet, and the slant height 15 feet: what is the convex surface of the cone?

Ans. 70.686 *sq. ft.*

4. The diameter of the base of a cone is 4,5 feet, and the slant height 20 feet: what is the entire surface?

Ans. 157.27635 *sq. ft.*

5. The circumference of the base of a cone is 10.75, and the slant height is 18.25: what is the entire surface?

Ans. 107.29021 *sq. ft.*

PROBLEM IV.

To find the solidity of a cone.

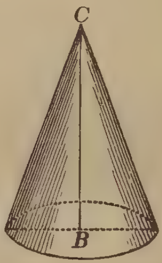
RULE.

Multiply the area of the base by the altitude; and divide the product by 3, *the quotient will be the solidity* (Bk. VI. Th. xviii).

Mensuration of the Round Bodies.

EXAMPLES.

1. What is the solidity of a cone, the area of whose base is 380 square feet, and altitude *CB*, 48 feet?



Operation.

We simply multiply the area of the base by the altitude, and then divide the product by 3.

```
      380
       48
     3040
    1520
   3)18240
area=6080
```

2. Required the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet.

Ans. 706.86 *cubic ft.*

3. Required the solidity of a cone whose altitude is $10\frac{1}{2}$ feet, and the circumference of its base 9 feet?

Ans. 22.5609 *cubic ft.*

4. What is the solidity of a cone, the diameter of whose base is 18 inches, and altitude 15 feet?

Ans. 8.83575 *cubic ft.*

5. The circumference of the base of a cone is 40 feet, and the altitude 50 feet: what is the solidity?

Ans. 2122.1333 *solid ft.*

PROBLEM V.

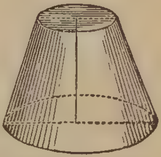
To find the surface of the frustum of a cone.

RULE.

Add together the circumferences of the two bases; and multiply the sum by half the slant height of the frustum; the product will be the convex surface, to which add the areas of the bases when the entire surface is required (Bk. VI. Th. ix).

EXAMPLES.

1. What is the convex surface of the frustum of a cone, of which the slant height is $12\frac{1}{2}$ feet, and the circumferences of the bases 8,4 and 6 feet.



We merely take the sum of the circumferences of the bases, and multiply by half the slant height, or side.

Operation.

8.4
6
14.4
half side 6.25
area = 90 *sq. ft.*

2. What is the entire surface of the frustum of a cone, the side being 16 feet, and the radii of the bases 2 and 3 feet?

Ans. 292.1688 *sq. ft.*

3. What is the convex surface of the frustum of a cone, the circumference of the greater base being 30 feet, and of the less 10 feet; the slant height being 20 feet?

Ans. 400 *sq. ft.*

4. Required the entire surface of the frustum of a cone whose slant height is 20 feet, and the diameters of the bases 8 and 4 feet

Ans. 439.824 *sq. ft.*

PROBLEM VI.

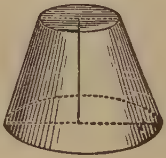
To find the solidity of the frustum of a cone

RULE.

I. *Add together the areas of the two ends and a geometrical mean between them.*

II. *Multiply this sum by one-third of the altitude and the product will be the solidity.*

EXAMPLES.



1. How many cubic feet in the frustum of a cone whose altitude is 26 feet, and the diameters of the bases 22 and 18 feet?

First, $\overline{22}^2 \times .7854 = 380.134 =$ area of lower base:

and $\overline{18}^2 \times .7854 = 254.47 =$ area of upper base.

Then, $\sqrt{380.134 \times 254.47} = 311.018 =$ mean.

Then, $(380.134 + 254,47 + 311.018) \times \frac{26}{3} = 8195.39$ which is the solidity.

2. How many cubic feet in a piece of round timber the diameter of the greater end being 18 inches, and that of the less 9 inches, and the length 14.25 feet? *Ans.* 14.68943.

3. What is the solidity of a frustum, the altitude being 18. the diameter of the lower base 8, and of the upper 4?

Ans. 527.7888.

4. If a cask, which is composed of two equal conic frustums joined together at their larger bases, have its bung diameter 28 inches, the head diameter 20 inches, and the length

40 inches, how many gallons of wine will it contain, there being 231 cubic inches in a gallon? *Ans.* 79.0613.

PROBLEM VII.

To find the surface of a sphere.

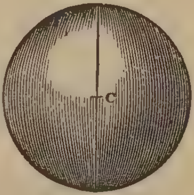
RULE.

Multiply the circumference of a great circle by the diameter, and the product will be the surface (Bk. VI. Th. xxiii).

EXAMPLES.

1. What is the surface of the sphere whose centre is C, the diameter being 7 feet?

Ans. 153.9384 *sq. ft.*



2. What is the surface of a sphere whose diameter is 24?

Ans. 1809.5616.

3. Required the surface of a sphere whose diameter is 7921 miles. *Ans.* 197111024 *sq. miles.*

4. What is the surface of a sphere the circumference of whose great circle is 78.54? *Ans.* 1963.5.

5. What is the surface of a sphere whose diameter is $1\frac{1}{3}$ feet? *Ans.* 5.58506 *sq. ft.*

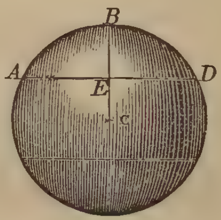
PROBLEM VIII.

To find the convex surface of a spherical zone.

RULE.

Multiply the height of the zone by the circumference of a great circle of the sphere, and the product will be the convex surface (Bk. VI. Th. xxiv)

EXAMPLES.



1. What is the convex surface of the zone *ABD*, the height *BE* being 9 inches, and the diameter of the sphere 42 inches?

First, $42 \times 3.1416 = 131.9472$ = circumference.
height = 9
surface = 1187.5248 square inches.

2. The diameter of a sphere is $12\frac{1}{2}$ feet: what will be the surface of a zone whose altitude is 2 feet?

Ans. 78.54 *sq. ft.*

3. The diameter of a sphere is 21 inches: what is the surface of a zone whose height is $4\frac{1}{2}$ inches?

Ans. 296.8812 *sq. in.*

4. The diameter of a sphere is 25 feet and the height of the zone 4 feet: what is the surface of the zone?

Ans. 314.16 *sq. ft.*

5. The diameter of a sphere is 9, and the height of a zone 3 feet: what is the surface of the zone?

Ans. 84.8232.

PROBLEM IX.

To find the solidity of a sphere.

RULE I.

Multiply the surface by one-third of the radius and the product will be the solidity (Bk. VI. Th. xxv).

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 12 feet?

First, 3.1416×12=37.6992= circumference of sphere.

diameter = 12

surface =452.3904

one-third radius = 2

Solidity =904.7808 cubic feet.

2. The diameter of a sphere is 7957.8: what is its solidity?
Ans. 263863122758.4778.

3. The diameter of a sphere is 24 yards: what is its solid contents? *Ans.* 7238.2464 *cubic yds.*

4. The diameter of a sphere is 8: what is its solidity?
Ans. 268.0832.

5. The diameter of a sphere is 16: what is its solidity?
Ans. 2144.6656.

RULE II.

Cube the diameter and multiply the number thus found, by the decimal .5236, *and the product will be the solidity.*

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 20?
Ans. 4188.8.

2 What is the solidity of a sphere whose diameter is 6?
Ans. 113.0976.

3. What is the solidity of a sphere whose diameter is 10?
Ans 523.6

PROBLEM X.

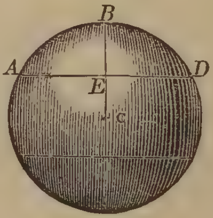
To find the solidity of a spherical segment with one base.

RULE.

I. *To three times the square of the radius of the base, add the square of the height.*

II. *Multiply this sum by the height, and the product by the decimal* .5236, *the result will be the solidity of the segment.*

EXAMPLES.



1. What is the solidity of the segment *ABD*, the height *BE* being 4 feet, and the diameter *AD* of the base being 14 feet?

First,

$$\overline{7}^2\times3+\overline{4}^2=147+16=163:$$

Then, $163\times4\times.5236=341.3872$ solid feet, which is the solidity of the segment.

2. What is the solidity of the segment of a sphere whose height is 4, and the radius of its base 8? *Ans.* 435.6352.

3. What is the solidity of a spherical segment, the diameter of its base being 17.23368, and its height 4.5?

Ans. 572.5566.

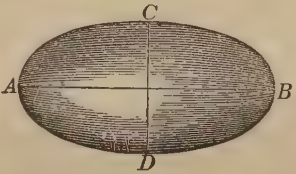
4. What is the solidity of a spherical segment, the diameter of the sphere being 8, and the height of the segment 2 feet? *Ans.* 41.888 *cubic ft.*

5. What is the solidity of a segment, when the diameter of the sphere is 20, and the altitude of the segment 9 feet?

Ans. 1781.2872 *cubic ft*

OF THE SPHEROID.

A spheroid is a solid described by the revolution of an ellipse about either of its axes.



If an ellipse $ACBD$, be revolved about the transverse or longer axis AB, the solid described is called a *prolate spheroid:* and if it be revolved about the shorter axis CD, the solid described is called an *oblate spheroid.*

The earth is an oblate spheroid, the axis about which it revolves being about 34 miles shorter than the diameter perpendicular to it.

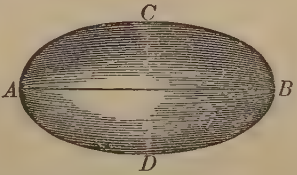
PROBLEM XI.

To find the solidity of an ellipsoid

RULE.

Multiply the fixed axis by the square of the revolving axis, and the product by the decimal .5236, *the result will be the required solidity.*

EXAMPLES.



1. In the prolate spheroid $ACBD$, the transverse axis $AB=90$, and the revolving axis $CD=70$ feet: what is the solidity?

Here, $AB=90$ feet: $\overline{CD}^2=\overline{70}^2=4900$: hence $AB\times\overline{CD}^2\times.5236=90\times4900\times.5236=230907.6$ cubic feet, which is the solidity.

2. What is the solidity of a prolate spheriod, whose fixed axis is 100, and revolving axis 6 feet? *Ans.* 1884.96.

3. What is the solidity of an oblate spheroid, whose fixed axis is 60, and revolving axis 100? *Ans.* 314160.

4. What is the solidity of a prolate spheroid, whose axes are 40 and 50? *Ans.* 41888.

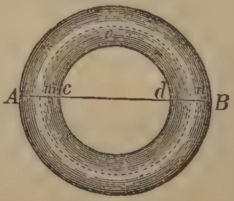
5. What is the solidity of an oblate spheroid, whose axes are 20 and 10? *Ans.* 2094.4.

6. What is the solidity of a prolate spheroid, whose axes are 55 and 33? *Ans.* 31361.022.

7. What is the solidity of an oblate spheroid, whose axes are 85 and 75? *Ans.* ——

OF CYLINDRICAL RINGS.

A cylindrical ring is formed by bending a cylinder until the two ends meet each other. Thus, if a cylinder be bent round until the axis takes the position *mon,* a solid will be formed, which is called a cylindrical ring.



The line *AB* is called the outer, and *cd* the inner diameter.

PROBLEM XII.

To find the convex surface of a cylindrical ring.

RULE.

I. *To the thickness of the ring add the inner diameter.*

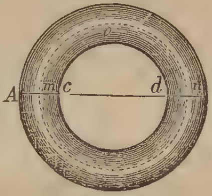
II. *Multiply this sum by the thickness, and the product by* 9.8696, *the result will be the area.*

EXAMPLES.

1. The thickness Ac, of a cylindrical ring is 3 inches, and the inner diameter cd, is 12 inches: what is the convex surface?

$Ac+cd=3+12=15$:

$15\times3\times9.8696=444.132$ square inches $=$ the surface.



2. The thickness of a cylindrical ring is 4 inches, and the inner diameter 18 inches: what is the convex surface?

Ans. 868.52 *sq. in.*

3. The thickness of a cylindrical ring is 2 inches, and the inner diameter 18 inches: what is the convex surface?

Ans. 394.784 *sq. in.*

PROBLEM XIII.

To find the solidity of a cylindrical ring.

RULE.

I *To the thickness of a ring add the inner diameter*

II. *Multiply this sum by the square of half the thickness, and the product by* 9.8696, *the result will be the required solidity.*

EXAMPLES.

1. What is the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches?

$8+3=11$: then, $11\times(\frac{3}{2})^2\times9.8696=244.2726$. which expresses the solidity in cubic inches.

2. The inner diameter of a cylindrical ring is 18 inches and the thickness 4 inches: what is the solidity of the ring?

Ans. 868.5248 *cubic inches*

3. Required the solidity of a cylindrical ring whose thickness is 2 inches, and inner diameter 12 inches?

Ans. 138.1744 *cubic in*

4. What is the solidity of a cylindrical ring, whose thickness is 4 inches, and inner diameter 16 inches?

Ans. 789.568 *cubic in.*

5. What is the solidity of a cylindrical ring, whose thickness is 8 inches, and inner diameter 20 inches?

Ans. ——

6. What is the solidity of a cylindrical ring whose thickness is 5 inches, and inner diameter 18 inches?

Ans. ——

A TABLE

OF

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0·000000	26	1·414973	51	1·707570	76	1·880814
2	0·301030	27	1·431364	52	1·716003	77	1·886491
3	0·477121	28	1·447158	53	1·724276	78	1·892095
4	0·602060	29	1·462398	54	1·732394	79	1·897627
5	0·698970	30	1·477121	55	1·740363	80	1·903090
6	0·778151	31	1·491362	56	1·748188	81	1·908485
7	0·845098	32	1·505150	57	1·755875	82	1·913814
8	0·903090	33	1·518514	58	1·763428	83	1·919078
9	0·954243	34	1·531479	59	1·770852	84	1·924279
10	1·000000	35	1·544068	60	1·778151	85	1·929419
11	1·041393	36	1·556303	61	1·785330	86	1·934498
12	1·079181	37	1·568202	62	1·792392	87	1·939519
13	1·113943	38	1·579784	63	1·799341	88	1·944483
14	1·146128	39	1·591065	64	1·806181	89	1·949390
15	1·176091	40	1·602060	65	1·812913	90	1·954243
16	1·204120	41	1·612784	66	1·819544	91	1·959041
17	1·230449	42	1·623249	67	1·826075	92	1·963788
18	1·255273	43	1·633468	68	1·832509	93	1·968483
19	1·278754	44	1·643453	69	1·838849	94	1·973128
20	1·301030	45	1·653213	70	1·845098	95	1·977724
21	1·322219	46	1·662758	71	1·851258	96	1·982271
22	1·342423	47	1·672098	72	1·857333	97	1·986772
23	1·361728	48	1·681241	73	1·863323	98	1·991226
24	1·380211	49	1·690196	74	1·869232	99	1·995635
25	1·397940	50	1·698970	75	1·875061	100	2·000000

REMARK.—In the following table, in the nine right-hand columns of each page, where the first or leading figures change from 9's to 0's, points or dots are introduced instead of the 0's, to catch the eye, and to indicate that from thence the two figures of the Logarithm to be taken from the second column, stand in the next line below.

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	•300	•724	1147	1570	1993	2415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	7033	7451	7868	8284	8700	9116	9532	9947	•361	•775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	•195	•600	1004	1408	1812	2216	2619	3021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	•207	•602	•998	396
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993	•380	•766	1153	1538	1924	2309	2694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	•320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	8186	8557	8928	9298	9668	••38	•407	•776	1145	1514	369
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	•266	•626	•987	1347	1707	2067	2426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	•258	•611	•963	1315	1667	2018	2370	2721	3071	351
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	••26	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	•253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	•245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	••12	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	9879	•194	•508	•822	1136	1450	1763	2076	2389	2702	314
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	•142	•449	•756	1063	1370	1676	1982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	•168	•469	•769	1068	301
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839	•126	•413	•699	•985	1272	1558	287
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	••51	281
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5899	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	••29	•303	•577	•850	1124	274
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
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N.	0	1	2	3	4	5	6	7	8	9	D.
160	204120	4391	4663	4934	5204	5475	5746	6016	6285	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	••51	•319	•586	•853	1121	1388	1654	1921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	•193	256
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	••50	•300	250
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	•176	245
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	•213	•446	•679	•912	1144	1377	1609	233
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	8982	9211	9439	9667	9895	•123	•351	•578	•806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	•161	•378	•595	•813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	••56	•268	•481	•693	•906	1118	1330	1542	212
205	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	322219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	••08	•211	203
214	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	••47	•246	199
219	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
220	342423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	••54	194
224	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835	••25	•215	•404	•593	•783	•972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	•143	•328	•513	•698	•883	185
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	••30	181
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	9166	9343	9520	9698	9875	••51	•228	•405	•582	•759	177
246	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	9674	9847	••20	•192	•365	•538	•711	•883	1056	1228	173
252	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933	•102	•271	•440	•609	•777	•946	1114	1283	1451	169
258	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	9956	•121	•286	•451	•616	•781	•945	1110	1275	1439	165
264	421604	1788	1933	2097	2261	2426	2590	2754	2918	3082	164
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594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
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653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
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668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
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676	9947	●●11	●●75	●139	●204	●268	●332	●396	●460	●525	64
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690	838849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
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704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
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706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
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894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
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936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	●●28	●●72	●117	●161	●206	●250	●294	44
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

A TABLE

OF

LOGARITHMIC

SINES AND TANGENTS

FOR EVERY

DEGREE AND MINUTE

OF THE QUADRANT.

REMARK. The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	0·000000		10·000000		0·000000		Infinite.	60
1	6·463726	5017·17	000000	·00	6·463726	5017·17	13·536274	59
2	764756	2934·85	000000	·00	764756	2934·83	235244	58
3	940847	2082·31	000000	·00	940847	2082·31	059153	57
4	7·065786	1615·17	000000	·00	7·065786	1615·17	12·934214	56
5	162696	1319·68	000000	·00	162696	1319·69	837304	55
6	241877	1115·75	9·999999	·01	241878	1115·78	758122	54
7	308824	966·53	999999	·01	308825	996·53	691175	53
8	366816	852·54	999999	·01	366817	852·54	633183	52
9	417968	762·63	999999	·01	417970	762·63	582030	51
10	463725	689·88	999998	·01	463727	689·88	536273	50
11	7·505118	629·81	9·999998	·01	7·505120	629·81	12·494880	49
12	542906	579·36	999997	·01	542909	579·33	457091	48
13	577668	536·41	999997	·01	577672	536·42	422328	47
14	609853	499·38	999996	·01	609857	499·39	390143	46
15	639816	467·14	999996	·01	639820	467·15	360180	45
16	667845	438·81	999995	·01	667849	438·82	332151	44
17	694173	413·72	999995	·01	694179	413·73	305821	43
18	718997	391·35	999994	·01	719004	391·36	280997	42
19	742477	371·27	999993	·01	742484	371·28	257516	41
20	764754	353·15	999993	·01	764761	351·36	235239	40
21	7·785943	336·72	9·999992	·01	7·785951	336·73	12·214049	39
22	806146	321·75	999991	·01	806155	321·76	193845	38
23	825451	308·05	999990	·01	825460	308·06	174540	37
24	843934	295·47	999989	·02	843944	295·49	156056	36
25	861662	283·88	999988	·02	861674	283·90	138326	35
26	878695	273·17	999988	·02	878708	273·18	121292	34
27	895085	263·23	999987	·02	895099	263·25	104901	33
28	910879	253·99	999986	·02	910894	254·01	089106	32
29	926119	245·38	999985	·02	926134	245·40	073866	31
30	940842	237·33	999983	·02	940858	237·35	059142	30
31	7·955082	229·80	9·999982	·02	7·955100	229·81	12·044900	29
32	968870	222·73	999981	·02	968889	222·75	031111	28
33	982233	216·08	999980	·02	982253	216·10	017747	27
34	995198	209·81	999979	·02	995219	209·83	004781	26
35	8·007787	203·90	999977	·02	8·007809	203·92	11·992191	25
36	020021	198·31	999976	·02	020045	198·33	979955	24
37	031919	193·02	999975	·02	031945	193·05	968055	23
38	043501	188·01	999973	·02	043527	188·03	956473	22
39	054781	183·25	999972	·02	054809	183·27	945191	21
40	065776	178·72	999971	·02	065806	178·74	934194	20
41	8·076500	174·41	9·999969	·02	8·076531	174·44	11·923469	19
42	086965	170·31	999968	·02	086997	170·34	913003	18
43	097183	166·39	999966	·02	097217	166·42	902783	17
44	107167	162·65	999964	·03	107202	162·68	892797	16
45	116926	159·08	999963	·03	116963	159·10	883037	15
46	126471	155·66	999961	·03	126510	155·68	873490	14
47	135810	152·38	999959	·03	135851	152·41	864149	13
48	144953	149·24	999958	·03	144996	149·27	855004	12
49	153907	146·22	999956	·03	153952	146·27	846048	11
50	162681	143·33	999954	·03	162727	143·36	837273	10
51	8·171280	140·54	9·999952	·03	8·171328	140·57	11·828672	9
52	179713	137·86	999950	·03	179763	137·90	820237	8
53	187985	135·29	999948	·03	188036	135·32	811964	7
54	196102	132·80	999946	·03	196156	132·84	803844	6
55	204070	130·41	999944	·03	204126	130·44	795874	5
56	211895	128·10	999942	·04	211953	128·14	788047	4
57	219581	125·87	999940	·04	219641	125·90	780359	3
58	227134	123·72	999938	·04	227195	123·76	772805	2
59	234557	121·64	999936	·04	234621	121·68	765379	1
60	241855	119·63	999934	·04	241921	119·67	758079	0
	Cosine	D.	Sine	89°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·241855	119·63	9·999934	·04	8·241921	119·67	11·758079	60
1	249033	117·68	999932	·04	249102	117·72	750898	59
2	256094	115·80	999929	·04	256165	115·84	743835	58
3	263042	113·98	999927	·04	263115	114·02	736885	57
4	269881	112·21	999925	·04	269956	112·25	730044	56
5	276614	110·50	999922	·04	276691	110·54	723309	55
6	283243	108·83	999920	·04	283323	108·87	716677	54
7	289773	107·21	999918	·04	289856	107·26	710144	53
8	296207	105·65	999915	·04	296292	105·70	703708	52
9	302546	104·13	999913	·04	302634	104·18	697366	51
10	308794	102·66	999910	·04	308884	102·70	691116	50
11	8·314904	101·22	9·999907	·04	8·315046	101·26	11·684954	49
12	321027	99·82	999905	·04	321122	99·87	678878	48
13	327016	98·47	999902	·04	327114	98·51	672886	47
14	332924	97·14	999899	·05	333025	97·19	666975	46
15	338753	95·86	999897	·05	338856	95·90	661144	45
16	344504	94·60	999894	·05	344610	94·65	655390	44
17	350181	93·38	999891	·05	350289	93·43	649711	43
18	355783	92·19	999888	·05	355895	92·24	644105	42
19	361315	91·03	999885	·05	361430	91·08	638570	41
20	366777	89·90	999882	·05	366895	89·95	633105	40
21	8·372171	88·80	9·999879	·05	8·372292	88·85	11·627708	39
22	377499	87·72	999876	·05	377622	87·77	622378	38
23	382762	86·67	999873	·05	382889	86·72	617111	37
24	387962	85·64	999870	·05	388092	85·70	611908	36
25	393101	84·64	999867	·05	393234	84·70	606766	35
26	398179	83·66	999864	·05	398315	83·71	601685	34
27	403199	82·71	999861	·05	403338	82·76	596662	33
28	408161	81·77	999858	·05	408304	81·82	591696	32
29	413068	80·86	999854	·05	413213	80·91	586787	31
30	417919	79·96	999851	·06	418068	80·02	581932	30
31	8·422717	79·09	9·999848	·06	8·422869	79·14	11·577131	29
32	427462	78·23	999844	·06	427618	78·30	572382	28
33	432156	77·40	999841	·06	432315	77·45	567685	27
34	436800	76·57	999838	·06	436962	76·63	563038	26
35	441394	75·77	999834	·06	441560	75·83	558440	25
36	445941	74·99	999831	·06	446110	75·05	553890	24
37	450440	74·22	999827	·06	450613	74·28	549387	23
38	454893	73·46	999823	·06	455070	73·52	544930	22
39	459301	72·73	999820	·06	459481	72·79	540519	21
40	463665	72·00	999816	·06	463849	72·06	536151	20
41	8·467985	71·29	9·999812	·06	8·468172	71·35	11·531828	19
42	472263	70·60	999809	·06	472454	70·66	527546	18
43	476498	69·91	999805	·06	476693	69·98	523307	17
44	480693	69·24	999801	·06	480892	69·31	519108	16
45	484848	68·59	999797	·07	485050	68·65	514950	15
46	488963	67·94	999793	·07	489170	68·01	510830	14
47	493040	67·31	999790	·07	493250	67·38	506750	13
48	497078	66·69	999786	·07	497293	66·76	502707	12
49	501080	66·08	999782	·07	501298	66·15	498702	11
50	505045	65·48	999778	·07	505267	65·55	494733	10
51	8·508974	64·89	9·999774	·07	8·509200	64·96	11·490800	9
52	512867	64·31	999769	·07	513098	64·39	486902	8
53	516726	63·75	999765	·07	516961	63·82	483039	7
54	520551	63·19	999761	·07	520790	63·26	479210	6
55	524343	62·64	999757	·07	524586	62·72	475414	5
56	528102	62·11	999753	·07	528349	62·18	471651	4
57	531828	61·58	999748	·07	532080	61·65	467920	3
58	535523	61·06	999744	·07	535779	61·13	464221	2
59	539186	60·55	999740	·07	539447	60·62	460553	1
60	542819	60·04	999735	·07	543084	60·12	456916	0
	Cosine	D.	Sine	88°	Cotang.	D.	Tang	

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·542819	60·04	9·999735	·07	8·543084	60·12	11·456916	60
1	546422	59·55	999731	·07	546691	59·62	453309	59
2	549995	59·06	999726	·07	550268	59·14	449732	58
3	553539	58·58	999722	·08	553817	58·66	446183	57
4	557054	58·11	999717	·08	557336	58·19	442664	56
5	560540	57·65	999713	·08	560828	57·73	439172	55
6	563999	57·19	999708	·08	564291	57·27	435709	54
7	567431	56·74	999704	·08	567727	56·82	432273	53
8	570836	56·30	999699	·08	571137	56·38	428863	52
9	574214	55·87	999694	·08	574520	55·95	425480	51
10	577566	55·44	999689	·08	577877	55·52	422123	50
11	8·580892	55·02	9·999685	·08	8·581208	55·10	11·418792	49
12	584193	54·60	999680	·08	584514	54·68	415486	48
13	587469	54·19	999675	·08	587795	54·27	412205	47
14	590721	53·79	999670	·08	591051	53·87	408949	46
15	593948	53·39	999665	·08	594283	53·47	405717	45
16	597152	53·00	999660	·08	597492	53·08	402508	44
17	600332	52·61	999655	·08	600677	52·70	399323	43
18	603489	52·23	999650	·08	603839	52·32	396161	42
19	606623	51·86	999645	09	606978	51·94	393022	41
20	609734	51·49	999640	·09	610094	51·58	389906	40
21	8·612823	51·12	9·999635	·09	8·613189	51·21	11·386811	39
22	615891	50·76	999629	·09	616262	50·85	383738	38
23	618937	50·41	999624	·09	619313	50·50	380687	37
24	621962	50·06	999619	·09	622343	50·15	377657	36
25	624965	49·72	999614	·09	625352	49·81	374648	35
26	627948	49·38	999608	·09	628340	49·47	371660	34
27	630911	49·04	999603	·09	631308	49·13	368692	33
28	633854	48·71	999597	·09	634256	48·80	365744	32
29	636776	48·39	999592	·09	637184	48·48	362816	31
30	639680	48·06	999586	·09	640093	48·16	359907	30
31	8·642563	47·75	9·999581	·09	8·642982	47·84	11·357018	29
32	645428	47·43	999575	·09	645853	47·53	354147	28
33	648274	47·12	999570	·09	648704	47·22	351296	27
34	651102	46·82	999564	·09	651537	46·91	348463	26
35	653911	46·52	999558	·10	654352	46·61	345648	25
36	656702	46·22	999553	·10	657149	46·31	342851	24
37	659475	45·92	999547	·10	659928	46·02	340072	23
38	662230	45·63	999541	·10	662689	45·73	337311	22
39	664968	45·35	999535	·10	665433	45·44	334567	21
40	667689	45·06	999529	·10	668160	45·26	331840	20
41	8·670393	44·79	9·999524	·10	8·670870	44·88	11·329130	19
42	673080	44·51	999518	·10	673563	44·61	326437	18
43	675751	44·24	999512	·10	676239	44·34	323761	17
44	678405	43·97	999506	·10	678900	44·17	321100	16
45	681043	43·70	999500	·10	681544	43·80	318456	15
46	683665	43·44	999493	·10	684172	43·54	315828	14
47	686272	43·18	999487	·10	686784	43·28	313216	13
48	688863	42·92	999481	·10	689381	43·03	310619	12
49	691438	42·67	999475	·10	691963	42·77	308037	11
50	693998	42·42	999469	·10	694529	42·52	305471	10
51	8·696543	42·17	9·999463	·11	8·697081	42·28	11·302919	9
52	699073	41·92	999456	·11	699617	42·03	300383	8
53	701589	41·68	999450	·11	702139	41·79	297861	7
54	704090	41·44	999443	·11	704646	41·55	295354	6
55	706577	41·21	999437	·11	707140	41·32	292860	5
56	709049	40·97	999431	·11	709618	41·08	290382	4
57	711507	40·74	999424	·11	712083	40·85	287917	3
58	713952	40·51	999418	·11	714534	40·62	285465	2
59	716383	40·29	999411	·11	716972	40·40	283028	1
60	718800	40·06	999404	·11	719396	40·17	280604	0
	Cosine	D.	Sine	87°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·718800	40·06	9·999404	·11	8·719396	40·17	11·280604	60
1	721204	39·84	999398	·11	721806	39·95	278194	59
2	723595	39·62	999391	·11	724204	39·74	275796	58
3	725972	39·41	999384	·11	726588	39·52	273412	57
4	728337	39·19	999378	·11	728959	39·30	271041	56
5	730688	38·98	999371	·11	731317	39·09	268683	55
6	733027	38·77	999364	·12	733663	38·89	266337	54
7	735354	38·57	999357	·12	735996	38·68	264004	53
8	737667	38·36	999350	·12	738317	38·48	261683	52
9	739969	38·16	999343	·12	740626	38·27	259374	51
10	742259	37·96	999336	·12	742922	38·07	257078	50
11	8·744536	37·76	9·999329	·12	8·745207	37·87	11·254793	49
12	746802	37·56	999322	·12	747479	37·68	252521	48
13	749055	37·37	999315	·12	749740	37·49	250260	47
14	751297	37·17	999308	·12	751989	37·29	248011	46
15	753528	36·98	999301	·12	754227	37·10	245773	45
16	755747	36·79	999294	·12	756453	36·92	243547	44
17	757955	36·61	999286	·12	758668	36·73	241332	43
18	760151	36·42	999279	·12	760872	36·55	239128	42
19	762337	36·24	999272	·12	763065	36·36	236935	41
20	764511	36·06	999265	·12	765246	36·18	234754	40
21	8·766675	35·88	9·999257	·12	8·767417	36·00	11·232583	39
22	768828	35·70	999250	·13	769578	35·83	230422	38
23	770970	35·53	999242	·13	771727	35·65	228273	37
24	773101	35·35	999235	·13	773866	35·48	226134	36
25	775223	35·18	999227	·13	775995	35·31	224005	35
26	777333	35·01	999220	·13	778114	35·14	221886	34
27	779434	34·84	999212	·13	780222	34·97	219778	33
28	781524	34·67	999205	·13	782320	34·80	217680	32
29	783605	34·51	999197	·13	784408	34·64	215592	31
30	785675	34·31	999189	·13	786486	34·47	213514	30
31	8·787736	34·18	9·999181	·13	8·788554	34·31	11·211446	29
32	789787	34·02	999174	·13	790613	34·15	209387	28
33	791828	33·86	999166	·13	792662	33·99	207338	27
34	793859	33·70	999158	·13	794701	33·83	205299	26
35	795881	33·54	999150	·13	796731	33·68	203269	25
36	797894	33·39	999142	·13	798752	33·52	201248	24
37	799897	33·23	999134	·13	800763	33·37	199237	23
38	801892	33·08	999126	·13	802765	33·22	197235	22
39	803876	32·93	999118	·13	804758	33·07	195242	21
40	805852	32·78	999110	·13	806742	32·92	193258	20
41	8·807819	32·63	9·999102	·13	8·808717	32·78	11·191283	19
42	809777	32·49	999094	·14	810683	32·62	189317	18
43	811726	32·34	999086	·14	812641	32·48	187359	17
44	813667	32·19	999077	·14	814589	32·33	185411	16
45	815599	32·05	999069	·14	816529	32·19	183471	15
46	817522	31·91	999061	·14	818461	32·05	181539	14
47	819436	31·77	999053	·14	820384	31·91	179616	13
48	821343	31·63	999044	·14	822298	31·77	177702	12
49	823240	31·49	999036	·14	824205	31·63	175795	11
50	825130	31·35	999027	·14	826103	31·50	173897	10
51	8·827011	31·22	9·999019	·14	8·827992	31·36	11·172008	9
52	828884	31·08	999010	·14	829874	31·23	170126	8
53	830749	30·95	999002	·14	831748	31·10	168252	7
54	832607	30·82	998993	·14	833613	30·96	166387	6
55	834456	30·69	998984	·14	835471	30·83	164529	5
56	836297	30·56	998976	·14	837321	30·70	162679	4
57	838130	30·43	998967	·15	839163	30·57	160837	3
58	839956	30·30	998958	·15	840998	30·45	159002	2
59	841774	30·17	998950	·15	842825	30·32	157175	1
60	843585	30·00	998941	·15	844644	30·19	155356	0
	Cosine	D	Sine	86°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·843585	30·05	9·998941	·15	8·844644	30·19	11·155356	60
1	845387	29·92	998932	·15	846455	30·07	153545	59
2	847183	29·80	998923	·15	848260	29·95	151740	58
3	848971	29·67	998914	·15	850057	29·82	149943	57
4	850751	29·55	998905	·15	851846	29·70	148154	56
5	852525	29·43	998896	·15	853628	29·58	146372	55
6	854291	29·31	998887	·15	855403	29·46	144597	54
7	856049	29·19	998878	·15	857171	29·35	142829	53
8	857801	29·07	998869	·15	858932	29·23	141068	52
9	859546	28·96	998860	·15	860686	29·11	139314	51
10	861283	28·84	998851	·15	862433	29·00	137567	50
11	8·863014	28·73	9·998841	·15	8·864173	28·88	11·135827	49
12	864738	28·61	998832	·15	865906	28·77	134094	48
13	866455	28·50	998823	·16	867632	28·66	132368	47
14	868165	28·39	998813	·16	869351	28·54	130649	46
15	869868	28·28	998804	·16	871064	28·43	128936	45
16	871565	28·17	998795	·16	872770	28·32	127230	44
17	873255	28·06	998785	·16	874469	28·21	125531	43
18	874938	27·95	998776	·16	876162	28·11	123838	42
19	876615	27·86	998766	·16	877849	28·00	122151	41
20	878285	27·73	998757	·16	879529	27·89	120471	40
21	8·879949	27·63	9·998747	·16	8·881202	27·79	11 118798	39
22	881607	27·52	998738	·16	882869	27·68	117131	38
23	883258	27·42	998728	·16	884530	27·58	115470	37
24	884903	27·31	998718	·16	886185	27·47	113815	36
25	886542	27·21	998708	·16	887833	27·37	112167	35
26	888174	27·11	998699	·16	889476	27·27	110524	34
27	889801	27·00	998689	·16	891112	27·17	108888	33
28	891421	26·90	998679	·16	892742	27·07	107258	32
29	893035	26·80	998669	·17	894366	26·97	105634	31
30	894643	26·70	998659	·17	895984	26·87	104016	30
31	8·896246	26·60	9·998649	·17	8·897596	26·77	11·102404	29
32	897842	26·51	998639	·17	899203	26·67	100797	28
33	899432	26·41	998629	·17	900803	26·58	099197	27
34	901017	26·31	998619	·17	902398	26·48	097602	26
35	902596	26·22	998609	·17	903987	26·38	096013	25
36	904169	26·12	998599	·17	905570	26·29	094430	24
37	905736	26·03	998589	·17	907147	26·20	092853	23
38	907297	25·93	998578	·17	908719	26·10	091281	22
39	908853	25·84	998568	·17	910285	26·01	089715	21
40	910404	25·75	998558	·17	911846	25·92	088154	20
41	8·911949	25·66	9·998548	·17	8·913401	25·83	11·086599	19
42	913488	25·56	998537	·17	914951	25·74	085049	18
43	915022	25·47	998527	·17	916495	25·65	083505	17
44	916550	25·38	998516	·18	918034	25·56	081966	16
45	918073	25·29	998506	·18	919568	25·47	080432	15
46	919591	25·20	998495	·18	921096	25·38	078904	14
47	921103	25·12	998485	·18	922619	25·30	077381	13
48	922610	25·03	998474	·18	924136	25·21	075864	12
49	924112	24·94	998464	·18	925649	25·12	074351	11
50	925609	24·86	998453	·18	927156	25·03	072844	10
51	8·927100	24·77	9·998442	·18	8·928658	24·95	11·071342	9
52	928587	24·69	998431	·18	930155	24·86	069845	8
53	930068	24·60	998421	·18	931647	24·78	068353	7
54	931544	24·52	998410	·18	933134	24·70	066866	6
55	933015	24·43	998399	·18	934616	24·61	065384	5
56	934481	24·35	998388	·18	936093	24·53	063907	4
57	935942	24·27	998377	·18	937565	24·45	062435	3
58	937398	24·19	998366	·18	939032	24·37	060968	2
59	938850	24·11	998355	·18	940494	24·30	059506	1
60	940296	24·03	998344	·18	941952	24·21	058048	0
	Cosine	D.	Sine	85°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·940296	24·03	9·998344	·19	8·941952	24·21	11·058048	60
1	941738	23·94	998333	·19	943404	24·13	056596	59
2	943174	23·87	998322	·19	944852	24·05	055148	58
3	944606	23·79	998311	·19	946295	23·97	053705	57
4	946034	23·71	998300	·19	947734	23·90	052266	56
5	947456	23·63	998289	·19	949168	23·82	050832	55
6	948874	23·55	998277	·19	950597	23·74	049403	54
7	950287	23·48	998266	·19	952021	23·66	047979	53
8	951696	23·40	998255	·19	953441	23·60	046559	52
9	953100	23·32	998243	·19	954856	23·51	045144	51
10	954499	23·25	998232	·19	956267	23·44	043733	50
11	8·955894	23·17	9·998220	19	8·957674	23·37	11·042326	49
12	957284	23·10	998209	·19	959075	23·29	040925	48
13	958670	23·02	998197	·19	960473	23·23	039527	47
14	960052	22·95	998186	·19	961866	23·14	038134	46
15	961429	22·88	998174	·19	963255	23·07	036745	45
16	962801	22·80	998163	·19	964639	23·00	035361	44
17	964170	22·73	998151	·19	966019	22·93	033981	43
18	965534	22·66	998139	·20	967394	22·86	032606	42
19	966893	22·59	998128	·20	968766	22·79	031234	41
20	968249	22·52	998116	·20	970133	22·71	029867	40
21	8·969600	22·44	9·998104	·20	8·071496	22·65	11·028504	39
22	970947	22·38	998092	·20	972855	22·57	027145	38
23	972289	22·31	998080	·20	974209	22·51	025791	37
24	973628	22·24	998068	·20	975560	22·44	024440	36
25	974962	22·17	998056	·20	976906	22·37	023094	35
26	976293	22·10	998044	·20	978248	22·30	021752	34
27	977619	22·03	998032	·20	979586	22·23	020414	33
28	978941	21·97	998020	·20	980921	22·17	019079	32
29	980259	21·90	998008	·20	982251	22·10	017749	31
30	981573	21·83	997996	·20	983577	22·04	016423	30
31	8·982883	21·77	9·997985	·20	8·984899	21·97	11·015101	29
32	984189	21·70	997972	·20	986217	21·91	013783	28
33	985491	21·63	997959	·20	987532	21·84	012468	27
34	986789	21·57	997947	·20	988842	21·78	011158	26
35	988083	21·50	997935	·21	990149	21·71	009851	25
36	989374	21·44	997922	·21	991451	21·65	008549	24
37	990660	21·38	997910	·21	992750	21·58	007250	23
38	991943	21·31	997897	·21	994045	21·52	005955	22
39	993222	21·25	997885	·21	995337	21·46	004663	21
40	994497	21·19	997872	·21	996624	21·40	003376	20
41	8·995768	21·12	9·997860	·21	8·997908	21·34	11·002092	19
42	997036	21·06	997847	·21	999188	21·27	000812	18
43	998299	21·00	997835	·21	9·000465	21·21	10·999535	17
44	999560	20·94	997822	·21	001738	21·15	998262	16
45	9·000816	20·87	997809	·21	003007	21·09	996993	15
46	002069	20·82	997797	·21	004272	21·03	995728	14
47	003318	20·76	997784	·21	005534	20·97	994466	13
48	004563	20·70	997771	·21	006792	20·91	993208	12
49	005805	20·64	997758	·21	008047	20·85	991953	11
50	007044	20·58	997745	·21	009298	20·80	990702	10
51	9·008278	20·52	9·997732	·21	9·010546	20·74	10·989454	9
52	009510	20·46	997719	·21	011790	20·68	988210	8
53	010737	20·40	997706	·21	013031	20·62	986969	7
54	011962	20·34	997693	·22	014268	20·56	985732	6
55	013182	20·29	997680	·22	015502	20·51	984498	5
56	014400	20·23	997667	·22	016732	20·45	983268	4
57	015613	20·17	997654	·22	017959	20·40	982041	3
58	016824	20·12	997641	·22	019183	20·33	980817	2
59	018031	20·06	997628	·22	020403	20·28	979597	1
60	019235	20·00	997614	·22	021620	20·23	978380	0
	Cosine	D.	Sine	84°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·019235	20·00	9·997614	·22	9·021620	20·23	10·978380	60
1	020435	19·95	997601	·22	022834	20·17	977166	59
2	021632	19·89	997588	·22	024044	20·11	975956	58
3	022825	19·84	997574	·22	025251	20·06	974749	57
4	024016	19·78	997561	·22	026455	20·00	973545	56
5	025203	19·73	997547	·22	027655	19·95	972345	55
6	026386	19·67	997534	·23	028852	19·90	971148	54
7	027567	19·62	997520	·23	030046	19·85	969954	53
8	028744	19·57	997507	·23	031237	19·79	968763	52
9	029918	19·51	997493	·23	032425	19·74	967575	51
10	031089	19·47	997480	·23	033609	19·69	966391	50
11	9·032257	19·41	9·997466	·23	9·034791	19·64	10·965209	49
12	033421	19·36	997452	·23	035969	19·58	964031	48
13	034582	19·30	997439	·23	037144	19·53	962856	47
14	035741	19·25	997425	·23	038316	19·48	961684	46
15	036896	19·20	997411	·23	039485	19·43	960515	45
16	038048	19·15	997397	·23	040651	19·38	959349	44
17	039197	19·10	997383	·23	041813	19·33	958187	43
18	040342	19·05	997369	·23	042973	19·28	957027	42
19	041485	18·99	997355	·23	044130	19·23	955870	41
20	042625	18·94	997341	·23	045284	19·18	954716	40
21	9·043762	18·89	9·997327	·24	9·046434	19·13	10·953566	39
22	044895	18·84	997313	·24	047582	19·08	952418	38
23	046026	18·79	697299	·24	048727	19·03	951273	37
24	047154	18·75	997285	·24	049869	18·98	950131	36
25	048279	18·70	997271	·24	051008	18·93	948992	35
26	049400	18·65	997257	·24	052144	18·89	947856	34
27	050519	18·60	997242	·24	053277	18·84	946723	33
28	051635	18·55	997228	·24	054407	18·79	945593	32
29	052749	18·50	997214	·24	055535	18·74	944465	31
30	053859	18·45	997199	·24	056659	18·70	943341	30
31	9·054966	18·41	9·997185	·24	9·057781	18·65	10·942219	29
32	056071	18·36	997170	·24	058900	18·69	941100	28
33	057172	18·31	997156	·24	060016	18·55	939984	27
34	058271	18·27	997141	·24	061130	18·51	938870	26
35	059367	18·22	997127	·24	062240	18·46	937760	25
36	060460	18·17	997112	·24	063348	18·42	936652	24
37	061551	18·13	997098	·24	064453	18·37	935547	23
38	062639	18·08	997083	·25	065556	18·33	934444	22
39	063724	18·04	997068	·25	066655	18·28	933345	21
40	064806	17·99	997053	·25	067752	18·24	932248	20
41	9·065885	17·94	9·997039	·25	9·068846	18·19	10·931154	19
42	066962	17·90	997024	·25	069938	18·15	930062	18
43	068036	17·86	997009	·25	071027	18·10	928973	17
44	069107	17·81	996994	·25	072113	18·06	927887	16
45	070176	17·77	996979	·25	073197	18·02	926803	15
46	071242	17·72	996964	·25	074278	17·97	925722	14
47	072306	17·68	996949	·25	075356	17·93	924644	13
48	073366	17·63	996934	·25	076432	17·89	923568	12
49	074424	17·59	996919	·25	077505	17·84	922495	11
50	075480	17·55	996904	·25	078576	17·80	921424	10
51	9·076533	17·50	9·996889	·25	9·079644	17·76	10·920356	9
52	077583	17·46	996874	·25	080710	17·72	919290	8
53	078631	17·42	996858	·25	081773	17·67	918227	7
54	079676	17·38	996843	·25	082833	17·63	917167	6
55	080719	17·33	996828	·25	083891	17·59	916109	5
56	081759	17·29	996812	·26	084947	17·55	915053	4
57	082797	17·25	996797	·26	086000	17·51	914000	3
58	083832	17·21	996782	·26	087050	17·47	912950	2
59	084864	17·17	996766	·26	088098	17·43	911902	1
60	085894	17·13	996751	·26	089144	17·38	910856	0
	Cosine	D.	Sine	83°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·085894	17·13	9·996751	·26	9·089144	17·38	10·910856	60
1	086922	17·09	996735	·26	090187	17·34	909813	59
2	087947	17·04	996720	·26	091228	17·30	908772	58
3	088970	17·00	996704	·26	092266	17·27	907734	57
4	089990	16·96	996688	·26	093302	17·22	906698	56
5	091008	16·92	996673	·26	094336	17·19	905664	55
6	092024	16·88	996657	·26	095367	17·15	904633	54
7	093037	16·84	996641	·26	096395	17·11	903605	53
8	094047	16·80	996625	·26	097422	17·07	902578	52
9	095056	16·76	996610	·26	098446	17·03	901554	51
10	096062	16·73	996594	·26	099468	16·99	900532	50
11	9·097065	16·68	9·996578	·27	9·100487	16·95	10·899513	49
12	098066	16·65	996562	·27	101504	16·91	898496	48
13	099065	16·61	996546	·27	102519	16·87	897481	47
14	100062	16·57	996530	·27	103532	16·84	896468	46
15	101056	16·53	996514	·27	104542	16·80	895458	45
16	102048	16·49	996498	·27	105550	16·76	894450	44
17	103037	16·45	996482	·27	106556	16·72	893444	43
18	104025	16·41	996465	·27	107559	16·69	892441	42
19	105010	16·38	996449	·27	108560	16·65	891440	41
20	105992	16·34	996433	·27	109559	16·61	890441	40
21	9·106973	16·30	9·996417	·27	9·110556	16·58	10·889444	39
22	107951	16·27	996400	·27	111551	16·54	888449	38
23	108927	16·23	996384	·27	112543	16·50	887457	37
24	109901	16·19	996368	·27	113533	16·46	886467	36
25	110873	16·16	996351	·27	114521	16·43	885479	35
26	111842	16·12	996335	·27	115507	16·39	884493	34
27	112809	16·08	996318	·27	116491	16·36	883509	33
28	113774	16·05	990302	·28	117472	16·32	882528	32
29	114737	16·01	996285	·28	118452	16·29	881548	31
30	115698	15·97	996269	·28	119429	16·25	880571	30
31	9·116656	15·94	9·996252	·28	9·120404	16·22	10·879596	29
32	117613	15·90	996235	·28	121377	16·18	878623	28
33	118567	15·87	996219	·28	122348	16·15	877652	27
34	119519	15·83	996202	·28	123317	16·11	876683	26
35	120469	15·80	996185	·28	124284	16·07	875716	25
36	121417	15·76	996168	·28	125249	16·04	874751	24
37	122362	15·73	996151	·28	126211	16·01	873789	23
38	123306	15·69	996134	·28	127172	15·97	872828	22
39	124248	15·66	996117	·28	128130	15·94	871870	21
40	125187	15·62	996100	·28	129087	15·91	870913	20
41	9·126125	15·59	9·996083	·29	9·130041	15·87	10·869959	19
42	127060	15·56	996066	·29	130994	15·84	869006	18
43	127993	15·52	996049	·29	131944	15·81	868056	17
44	128925	15·49	996032	·29	132893	15·77	867107	16
45	129854	15·45	996015	·29	133839	15·74	866161	15
46	130781	15·42	995998	·29	134784	15·71	865216	14
47	131706	15·39	995980	·29	135726	15·67	864274	13
48	132630	15·35	995963	·29	136667	15·64	863333	12
49	133551	15·32	995946	·29	137605	15·61	862395	11
50	134470	15·29	995928	·29	138542	15·58	861458	10
51	9·135387	15·25	9·995911	·29	9·139476	15·55	10·860524	9
52	136303	15·22	995894	·29	140409	15·51	859591	8
53	137216	15·19	995876	·29	141340	15·48	858660	7
54	138128	15·16	995859	·29	142269	15·45	857731	6
55	139037	15·12	995841	·29	143196	15·42	856804	5
56	139944	15·09	995823	·29	144121	15·39	855879	4
57	140850	15·06	995806	·29	145044	15·35	854956	3
58	141754	15·03	995788	·29	145966	15·32	854034	2
59	142655	15·00	995771	·29	146885	15·29	853115	1
60	143555	14·96	995753	·29	147803	15·26	852197	0
	Cosine	D.	Sine	82°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·143555	14·96	9·995753	·30	9·147803	15·26	10·852197	60
1	144453	14·93	995735	·30	148718	15·23	851282	59
2	145349	14·90	995717	·30	149632	15·20	850368	58
3	146243	14·87	995699	·30	150544	15·17	849456	57
4	147136	14·84	995681	·30	151454	15·14	848546	56
5	148026	14·81	995664	·30	152363	15·11	847637	55
6	148915	14·78	995646	·30	153269	15·08	846731	54
7	149802	14·75	995628	·30	154174	15·05	845826	53
8	150686	14·72	995610	·30	155077	15·02	844923	52
9	151569	14·69	995591	·30	155978	14·99	844022	51
10	152451	14·66	995573	·30	156877	14·96	843123	50
11	9·153330	14·63	9·995555	·30	9·157775	14·93	10·842225	49
12	154208	14·60	995537	·30	158671	14·90	841329	48
13	155083	14·57	995519	·30	159565	14·87	840435	47
14	155957	14·54	995501	·31	160457	14·84	839543	46
15	156830	14·51	995482	·31	161347	14·81	838653	45
16	157700	14·48	995464	·31	162236	14·79	837764	44
17	158569	14·45	995446	·31	163123	14·76	836877	43
18	159435	14·42	995427	·31	164008	14·73	835992	42
19	160301	14·39	995409	·31	164892	14·70	835108	41
20	161164	14·36	995390	·31	165774	14·67	834226	40
21	9·162025	14·33	9·995372	·31	9·166654	14·64	10·833346	39
22	162885	14·30	995353	·31	167532	14·61	832468	38
23	163743	14·27	995334	·31	168409	14·58	831591	37
24	164600	14·24	995316	·31	169284	14·55	830716	36
25	165454	14·22	995297	·31	170157	14·53	829843	35
26	166307	14·19	995278	·31	171029	14·50	828971	34
27	167159	14·16	995260	·31	171899	14·47	828101	33
28	168008	14·13	995241	·32	172767	14·44	827233	32
29	168856	14·10	995222	·32	173634	14·42	826366	31
30	169702	14·07	995203	·32	174499	14·39	825501	30
31	9·170547	14·05	9·995184	·32	9·175362	14·36	10·824638	29
32	171389	14·02	995165	·32	176224	14·33	823776	28
33	172230	13·99	995146	·32	177084	14·31	822916	27
34	173070	13·96	995127	·32	177942	14·28	822058	26
35	173908	13·94	995108	·32	178799	14·25	821201	25
36	174744	13·91	995089	·32	179655	14·23	820345	24
37	175578	13·88	995070	·32	180508	14·20	819492	23
38	176411	13·86	995051	·32	181360	14·17	818640	22
39	177242	13·83	995032	·32	182211	14·15	817789	21
40	178072	13·80	995013	·32	183059	14·12	816941	20
41	9·178900	13·77	9·994993	·32	9·183907	14·09	10·816093	19
42	179726	13·74	994974	·32	184752	14·07	815248	18
43	180551	13·72	994955	·32	185597	14·04	814403	17
44	181374	13·69	994935	·32	186439	14·02	813561	16
45	182196	13·66	994916	·33	187280	13·99	812720	15
46	183016	13·64	994896	·33	188120	13·96	811880	14
47	183834	13·61	994877	·33	188958	13·93	811042	13
48	184651	13·59	994857	·33	189794	13·91	810206	12
49	185466	13·56	994838	·33	190629	13·89	809371	11
50	186280	13·53	994818	·33	191462	13·86	808538	10
51	9·187092	13·51	9·994798	·33	9·192294	13·84	10·807706	9
52	187903	13·48	994779	·33	193124	13·81	806876	8
53	188712	13·46	994759	·33	193953	13·79	806047	7
54	189519	13·43	994739	·33	194780	13·76	805220	6
55	190325	13·41	994719	·33	195606	13·74	804394	5
56	191130	13·38	994700	·33	196430	13·71	803570	4
57	191933	13·36	994680	·33	197253	13·69	802747	3
58	192734	13·33	994660	·33	198074	13·66	801926	2
59	193534	13·30	994640	·33	198894	13·64	801106	1
60	194332	13·28	994620	·33	199713	13·61	800287	0
	Cosine	D.	Sine	81°	Cotang.	D.	Tang.	M.

M	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·194332	13·28	9·994620	·33	9·199713	13·61	0·800287	60
1	195129	13·26	994600	·33	200529	13·59	799471	59
2	195925	13·23	994580	·33	201345	13·56	798655	58
3	196719	13·21	994560	·34	202159	13·54	797841	57
4	197511	13·18	994540	·34	202971	13·52	797029	56
5	198302	13·16	994519	·34	203782	13·49	796118	55
6	199091	13·13	994499	·34	204592	13·47	795408	54
7	199879	13·11	994479	·34	205400	13·45	794600	53
8	200666	13·08	994459	·34	206207	13·42	793793	52
9	201451	13·06	994438	·34	207013	13·40	792987	51
10	202234	13·04	994418	·34	207817	13·38	792183	50
11	9·203017	13·01	9·994397	·34	9·208619	13·35	10·791381	49
12	203797	12·99	994377	·34	209420	13·33	790580	48
13	204577	12·96	994357	·34	210220	13·31	789780	47
14	205354	12·94	994336	·34	211018	13·28	788982	46
15	206131	12·92	994316	·34	211815	13·26	788185	45
16	206906	12·89	994295	·34	212611	13·24	787389	44
17	207679	12·87	994274	·35	213405	13·21	786595	43
18	208452	12·85	994254	·35	214198	13·19	785802	42
19	209222	12·82	994233	·35	214989	13·17	785011	41
20	209992	12·80	994212	·35	215780	13·15	784220	40
21	9·210760	12·78	9·994191	·35	9·216568	13·12	10·783432	39
22	211526	12·75	994171	·35	217356	13·10	782644	38
23	212291	12·73	994150	·35	218142	13·08	781858	37
24	213055	12·71	994129	·35	218926	13·05	781074	36
25	213818	12·68	994108	·35	219710	13·03	780290	35
26	214579	12·66	994087	·35	220492	13·01	779508	34
27	215338	12·64	994066	·35	221272	12·99	778728	33
28	216097	12·61	994045	·35	222052	12·97	777948	32
29	216854	12·59	994024	·35	222830	12·94	777170	31
30	217609	12·57	994003	·35	223606	12·92	776394	30
31	9·218363	12·55	9·993981	·35	9·224382	12·90	10·775618	29
32	219116	12·53	993960	·35	225156	12·88	774844	28
33	219868	12·50	993939	·35	225929	12 86	774071	27
34	220618	12·48	993918	·35	226700	12 84	773300	26
35	221367	12·46	993896	·36	227471	12 81	772529	25
36	222115	12·44	993875	·36	228239	12·79	771761	24
37	222861	12·42	993854	·36	229007	12·77	770993	23
38	223606	12·39	993832	·36	229773	12·75	770227	22
39	224349	12·37	993811	·36	230539	12·73	769461	21
40	225092	12·35	993789	·36	231302	12·71	768698	20
41	9·225833	12·33	9·993768	·36	9·232065	12·69	10·767935	19
42	226573	12·31	993746	·36	232826	12·67	767174	18
43	227311	12·28	993725	·36	233586	12·65	766414	17
44	228048	12·26	993703	·36	234345	12·62	765655	16
45	228784	12·24	993681	·36	235103	12·60	764897	15
46	229518	12·22	993660	·36	235859	12·58	764141	14
47	230252	12·20	993638	·36	236614	12·56	763386	13
48	230984	12·18	993616	·36	237368	12·54	762632	12
49	231714	12·16	993594	·37	238120	12·52	761880	11
50	232444	12·14	993572	·37	238872	12·50	761128	10
51	9·233172	12·12	9·993550	·37	9·239622	12·48	10·760378	9
52	233899	12·09	993528	·37	240371	12·46	759629	8
53	234625	12·07	993506	·37	241118	12·44	758882	7
54	235349	12·05	993484	·37	241865	12·42	758135	6
55	236073	12·03	993462	·37	242610	12·40	757390	5
56	236795	12·01	993440	·37	243354	12·38	756646	4
57	237515	11·99	993418	·37	244097	12·36	755903	3
58	238235	11·97	993396	·37	244839	12·34	755161	2
59	238953	11·95	993374	·37	245579	12·32	754421	1
60	239670	11·93	993351	·37	246319	12·30	753681	0
	Cosine	D.	Sine	80°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·239670	11·93	9·993351	·37	9·246319	12·30	10·753681	60
1	240386	11·91	993329	·37	247057	12·28	752943	59
2	241101	11·89	993307	·37	247794	12·26	752206	58
3	241814	11·87	993285	·37	248530	12·24	751470	57
4	242526	11·85	993262	·37	249264	12·22	750736	56
5	243237	11·83	993240	·37	249998	12·20	750002	55
6	243947	11·81	993217	·38	250730	12·18	749270	54
7	244656	11·79	993195	·38	251461	12·17	748539	53
8	245363	11·77	993172	·38	252191	12·15	747809	52
9	246069	11·75	993149	·38	252920	12·13	747080	51
10	246775	11·73	993127	·38	253648	12·11	746352	50
11	9·247478	11·71	9·993104	·38	9·254374	12·09	10·745626	49
12	248181	11·69	993081	·38	255100	12·07	744900	48
13	248883	11·67	993059	·38	255824	12 05	744176	47
14	249583	11·65	993036	·38	256547	12·03	743453	46
15	250282	11·63	993013	·38	257269	12·01	742731	45
16	250980	11·61	992990	·38	257990	12·00	742010	44
17	251677	11·59	992967	·38	258710	11·98	741290	43
18	252373	11·58	992944	·38	259429	11·96	740571	42
19	253067	11·56	992921	·38	260146	11·94	739854	41
20	253761	11·54	992898	·38	260863	11·92	739137	40
21	9·254453	11·52	9·992875	·38	9·261578	11·90	10·738422	39
22	255144	11 50	992852	·38	262292	11·89	737708	38
23	255834	11·48	992829	·39	263005	11·87	736995	37
24	256523	11·46	992806	·39	263717	11·85	736283	36
25	257211	11·44	992783	·39	264428	11·83	735572	35
26	257898	11·42	992759	·39	265138	11·81	734862	34
27	258583	11·41	992736	·39	265847	11·79	734153	33
28	259268	11·39	992713	·39	266555	11·78	733445	32
29	259951	11·37	992690	·39	267261	11·76	732739	31
30	260633	11·35	992666	·39	267967	11·74	732033	30
31	9·261314	11·33	9·992643	·39	9·268671	11·72	10·731329	29
32	261994	11·31	992619	·39	269375	11·70	730625	28
33	262673	11·30	992596	·39	270077	11·69	729923	27
34	263351	11·28	992572	·39	270779	11·67	729221	26
35	264027	11·26	992549	·39	271479	11·65	728521	25
36	264703	11·24	992525	·39	272178	11·64	727822	24
37	265377	11·22	992501	·39	272876	11·62	727124	23
38	266051	11·20	992478	·40	273573	11·60	726427	22
39	266723	11·19	992454	·40	274269	11·58	725731	21
40	267395	11·17	992430	·40	274964	11·57	725036	20
41	9·268065	11·15	9·992406	·40	9·275658	11·55	10·724342	19
42	268734	11·13	992382	·40	276351	11·53	723649	18
43	269402	11·11	992359	·40	277043	11·51	722957	17
44	270069	11·10	992335	·40	277734	11·50	722266	16
45	270735	11·08	992311	·40	278424	11·48	721576	15
46	271400	11·06	992287	·40	279113	11·47	720887	14
47	272064	11·05	992263	·40	279801	11·45	720199	13
48	272726	11·03	992239	·40	280488	11·43	719512	12
49	273388	11·01	992214	·40	281174	11·41	718826	11
50	274049	10·99	992190	·40	281858	11·40	718142	10
51	9 274708	10·98	9·992166	·40	9·282542	11·38	10·717458	9
52	275367	10·96	992142	·40	283225	11·36	716775	8
53	276024	10·94	992117	·41	283907	11·35	716093	7
54	276681	10·92	992093	·41	284588	11·33	715412	6
55	277337	10·91	992069	·41	285268	11·31	714732	5
56	277991	10·89	992044	·41	285947	11·30	714053	4
57	278644	10·87	992020	·41	286624	11·28	713376	3
58	279297	10·86	991996	·41	287301	11·26	712699	2
59	279948	10·84	991971	·41	287977	11·25	712023	1
60	280599	10·82	991947	·41	288652	11·23	711348	0
	Cosine	D.	Sine	79°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·280599	10·82	9·991947	·41	9·288652	11·23	10·711348	60
1	281248	10·81	991922	·41	289326	11·22	710674	59
2	281897	10·79	991897	·41	289999	11·20	710001	58
3	282544	10·77	991873	·41	290671	11·18	709329	57
4	283190	10·76	991848	·41	291342	11·17	708658	56
5	283836	10·74	991823	·41	292013	11·15	707987	55
6	284480	10·72	991799	·41	292682	11·14	707318	54
7	285124	10·71	991774	·42	293350	11·12	706650	53
8	285766	10·69	991749	·42	294017	11·11	705983	52
9	286408	10·67	991724	·42	294684	11·09	705316	51
10	287048	10·66	991699	·42	295349	11·07	704651	50
11	9·287687	10·64	9·991674	·42	9·296013	11·06	10·703987	49
12	288326	10·63	991649	·42	296677	11·04	703323	48
13	288964	10·61	991624	·42	297339	11·03	702661	47
14	289600	10·59	991599	·42	298001	11·01	701999	46
15	290236	10·58	991574	·42	298662	11·00	701338	45
16	290870	10·56	991549	·42	299322	10·98	700678	44
17	291504	10·54	991524	·42	299980	10·96	700020	43
18	292137	10·53	991498	·42	300638	10·95	699362	42
19	292768	10·51	991473	·42	301295	10·93	698705	41
20	293399	10·50	991448	·42	301951	10·92	698049	40
21	9·294029	10·48	9·991422	·42	9·302607	10·90	10·697393	39
22	294658	10·46	991397	·42	303261	10·89	696739	38
23	295286	10·45	991372	·43	303914	10·87	696086	37
24	295913	10·43	991346	·43	304567	10·86	695433	36
25	296539	10·42	991321	·43	305218	10·84	694782	35
26	297164	10·40	991295	·43	305869	10·83	694131	34
27	297788	10·39	991270	·43	306519	10·81	693481	33
28	298412	10·37	991244	·43	307168	10·80	692832	32
29	299034	10·36	991218	·43	307815	10·78	692185	31
30	299655	10·34	991193	·43	308463	10·77	691537	30
31	9·300276	10·32	9·991167	·43	9·309109	10·75	10·690891	29
32	300895	10·31	991141	·43	309754	10·74	690246	28
33	301514	10·29	991115	·43	310398	10·73	689602	27
34	302132	10·28	991090	·43	311042	10·71	688958	26
35	302748	10·26	991064	·43	311685	10·70	688315	25
36	303364	10·25	991038	·43	312327	10·68	687673	24
37	303979	10·23	991012	·43	312967	10·67	687033	23
38	304593	10·22	990986	·43	313608	10·65	686392	22
39	305207	10·20	990960	·43	314247	10·64	685753	21
40	305819	10·19	990934	·44	314885	10·62	685115	20
41	9·306430	10·17	9·990908	·44	9·315523	10·61	10·684477	19
42	307041	10·16	990882	·44	316159	10·60	683841	18
43	307650	10·14	990855	·44	316795	10·58	683205	17
44	308259	10·13	990829	·44	317430	10·57	682570	16
45	308867	10·11	990803	·44	318064	10·55	681936	15
46	309474	10·10	990777	·44	318697	10·54	681303	14
47	310080	10·08	990750	·44	319329	10·53	680671	13
48	310685	10·07	990724	·44	319961	10·51	680039	12
49	311289	10·05	990697	·44	320592	10·50	679408	11
50	311893	10·04	990671	·44	321222	10·48	678778	10
51	9·312495	10·03	9·990644	·44	9·321851	10·47	10·678149	9
52	313097	10·01	990618	·44	322479	10·45	677521	8
53	313698	10·00	990591	·44	323106	10·44	676894	7
54	314297	9·98	990565	·44	323733	10·43	676267	6
55	314897	9·97	990538	·44	324358	10·41	675642	5
56	315495	9·96	990511	·45	324983	10·40	675017	4
57	316092	9·94	990485	·45	325607	10·39	674393	3
58	316689	9·93	990458	·45	326231	10·37	673769	2
59	317284	9·91	990431	·45	326853	10·36	673147	1
60	317879	9·90	990404	·45	327475	10·35	672525	0
	Cosine	D.	Sine	78°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·317879	9·90	9·990404	·45	9·327474	10·35	10·672526	60
1	318473	9·88	990378	·45	328095	10·33	671905	59
2	319066	9·87	990351	·45	328715	10·32	671285	58
3	319658	9·86	990324	·45	329334	10·30	670666	57
4	320249	9·84	990297	·45	329953	10·29	670047	56
5	320840	9·83	990270	·45	330570	10·28	669430	55
6	321430	9·82	990243	·45	331187	10·26	668813	54
7	322019	9·80	990215	·45	331803	10·25	668197	53
8	322607	9·79	990188	·45	332418	10·24	667582	52
9	323194	9·77	990161	·45	333033	10·23	666967	51
10	323780	9·76	990134	·45	333646	10·21	666354	50
11	9·324366	9·75	9·990107	·46	9·334259	10·20	10·665741	49
12	324950	9·73	990079	·46	334871	10·19	665129	48
13	325534	9·72	990052	·46	335482	10·17	664518	47
14	326117	9·70	990025	·46	336093	10·16	663907	46
15	326700	9·69	989997	·46	336702	10·15	663298	45
16	327281	9·68	989970	·46	337311	10·13	662689	44
17	327862	9·66	989942	·46	337919	10·12	662081	43
18	328442	9·65	989915	·46	338527	10·11	661473	42
19	329021	9·64	989887	·46	339133	10·10	660867	41
20	329599	9·62	989860	·46	339739	10·08	660261	40
21	9·330176	9·61	9·989832	·46	9·340344	10·07	10·659656	39
22	330753	9·60	989804	·46	340948	10·06	659052	38
23	331329	9·58	989777	·46	341552	10·04	658448	37
24	331903	9·57	989749	·47	342155	10·03	657845	36
25	332478	9·56	989721	·47	342757	10·02	657243	35
26	333051	9·54	989693	·47	343358	10·00	656642	34
27	333624	9·53	989665	·47	343958	9·99	656042	33
28	334195	9·52	989637	·47	344558	9·98	655442	32
29	334766	9·50	989609	·47	345157	9·97	654843	31
30	335337	9·49	989582	·47	345755	9·96	654245	30
31	9·335906	9·48	9·989553	·47	9·346353	9·94	10·653647	29
32	336475	9·46	989525	·47	346949	9·93	653051	28
33	337043	9·45	989497	·47	347545	9·92	652455	27
34	337610	9·44	989469	·47	348141	9·91	651859	26
35	338176	9·43	989441	·47	348735	9·90	651265	25
36	338742	9·41	989413	·47	349329	9·88	650671	24
37	339306	9·40	989384	·47	349922	9·87	650078	23
38	339871	9·39	989356	·47	350514	9·86	649486	22
39	340434	9·37	989328	·47	351106	9·85	648894	21
40	340996	9·36	989300	·47	351697	9·83	648303	20
41	9·341558	9·35	9·989271	·47	9·352287	9·82	10·647713	19
42	342119	9·34	989243	·47	352876	9·81	647124	18
43	342679	9·32	989214	·47	353465	9·80	646535	17
44	343239	9·31	989186	·47	354053	9·79	645947	16
45	343797	9·30	989157	·47	354640	9·77	645360	15
46	344355	9·29	989128	·48	355227	9·76	644773	14
47	344912	9·27	989100	·48	355813	9·75	644187	13
48	345469	9·26	989071	·48	356398	9·74	643602	12
49	346024	9·25	989042	·48	356982	9·73	643018	11
50	346579	9·24	989014	·48	357566	9·71	642434	10
51	9·347134	9·22	9·988985	·48	9·358149	9·70	10·641851	9
52	347687	9·21	988956	·48	358731	9·69	641269	8
53	348240	9·20	988927	·48	359313	9·68	640687	7
54	348792	9·19	988898	·48	359893	9·67	640107	6
55	349343	9·17	988869	·48	360474	9·66	639526	5
56	349893	9·16	988840	·48	361053	9·65	638947	4
57	350443	9·15	988811	·49	361632	9·63	638368	3
58	350992	9·14	988782	·49	362210	9·62	637790	2
59	351540	9·13	988753	·49	362787	9·61	637213	1
60	352088	9·11	988724	·49	363364	9·60	636636	0
	Cosine	D.	Sine	77°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·352088	9·11	9·988724	·49	9·363364	9·60	10·636636	60
1	352635	9·10	988695	·49	363940	9·59	636060	59
2	353181	9·09	988666	·49	364515	9·58	635485	58
3	353726	9·08	988636	·49	365090	9·57	634910	57
4	354271	9·07	988607	·49	365664	9·55	634336	56
5	354815	9·05	988578	·49	366237	9·54	633763	55
6	355358	9·04	988548	·49	366810	9·53	633190	54
7	355901	9·03	988519	·49	367382	9·52	632618	53
8	356443	9·02	988489	·49	367953	9·51	632047	52
9	356984	9·01	988460	·49	368524	9·50	631476	51
10	357524	8·99	988430	·49	369094	9·49	630906	50
11	9·358064	8·98	9·988401	·49	9·369663	9·48	10·630337	49
12	358603	8·97	988371	·49	370232	9·46	629768	48
13	359141	8·96	988342	·49	370799	9·45	629201	47
14	359678	8·95	988312	·50	371367	9·44	628633	46
15	360215	8·93	988282	·50	371933	9·43	628067	45
16	360752	8·92	988252	·50	372499	9·42	627501	44
17	361287	8·91	988223	·50	373064	9·41	626936	43
18	361822	8·90	988193	·50	373629	9·40	626371	42
19	362356	8·89	988163	·50	374193	9·39	625807	41
20	362889	8·88	988133	·50	374756	9·38	625244	40
21	9·363422	8·87	9·988103	·50	9·375319	9·37	10·624681	39
22	363954	8·85	988073	·50	375881	9·35	624119	38
23	364485	8·84	988043	·50	376442	9·34	623558	37
24	365016	8·83	988013	·50	377003	9·33	622997	36
25	365546	8·82	987983	·50	377563	9·32	622437	35
26	366075	8·81	987953	·50	378122	9·31	621878	34
27	366604	8·80	987922	·50	378681	9·30	621319	33
28	367131	8·79	987892	·50	379239	9·29	620761	32
29	367659	8·77	987862	·50	379797	9·28	620203	31
30	368185	8·76	987832	·51	380354	9·27	619646	30
31	9·368711	8·75	9·987801	·51	9·380910	9·26	10·619090	29
32	369236	8·74	987771	·51	381466	9·25	618534	28
33	369761	8·73	987740	·51	382020	9·24	617980	27
34	370285	8·72	987710	·51	382575	9·23	617425	26
35	370808	8·71	987679	·51	383129	9·22	616871	25
36	371330	8·70	987649	·51	383682	9·21	616318	24
37	371852	8·69	987618	·51	384234	9·20	615766	23
38	372373	8·67	987588	·51	384786	9·19	615214	22
39	372894	8·66	987557	·51	385337	9·18	614663	21
40	373414	8·65	987526	·51	385888	9·17	614112	20
41	9·373933	8·64	9·987496	·51	9·386438	9·15	10·613562	19
42	374452	8·63	987465	·51	386987	9·14	613013	18
43	374970	8·62	987434	·51	387536	9·13	612464	17
44	375487	8·61	987403	·52	388084	9·12	611916	16
45	376003	8·60	987372	·52	388631	9·11	611369	15
46	376519	8·59	987341	·52	389178	9·10	610822	14
47	377035	8·58	987310	·52	389724	9·09	610276	13
48	377549	8·57	987279	·52	390270	9·08	609730	12
49	378063	8·56	987248	·52	390815	9·07	609185	11
50	378577	8·54	987217	·52	391360	9·06	608640	10
51	9·379089	8·53	9·987186	·52	9·391903	9·05	10·608097	9
52	379601	8·52	987155	·52	392447	9·04	607553	8
53	380113	8·51	987124	·52	392989	9·03	607011	7
54	380624	8·50	987092	·52	393531	9·02	606469	6
55	381134	8·49	987061	·52	394073	9·01	605927	5
56	381643	8·48	987030	·52	394614	9·00	605386	4
57	382152	8·47	986998	·52	395154	8·99	604846	3
58	382661	8·46	986967	·52	395694	8·98	604306	2
59	383168	8·45	986936	·52	396233	8·97	603767	1
60	383675	8·44	986904	·52	396771	8·96	603229	0
	Cosine	D.	Sine	76°	Cotang.	D.	Tang	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·383675	8·44	9·986904	·52	9·396771	8·96	10·603229	60
1	384182	8·43	986873	·53	397309	8·96	602691	59
2	384687	8·42	986841	·53	397846	8·95	602154	58
3	385192	8·41	986809	·53	398383	8·94	601617	57
4	385697	8·40	986778	·53	398919	8·93	601081	56
5	386201	8·39	986746	·53	399455	8·92	600545	55
6	386704	8·38	986714	·53	399990	8·91	600010	54
7	387207	8·37	986683	·53	400524	8·90	599476	53
8	387709	8·36	986651	·53	401058	8·89	598942	52
9	388210	8·35	986619	·53	401591	8·88	598409	51
10	388711	8·34	986587	·53	402124	8·87	597876	50
11	9·389211	8·33	9·986555	·53	9·402656	8·86	10·597344	49
12	389711	8·32	986523	·53	403187	8·85	596813	48
13	390210	8·31	986491	·53	403718	8·84	596282	47
14	390708	8·30	986459	·53	404249	8·83	595751	46
15	391206	8·28	986427	·53	404778	8·82	595222	45
16	391703	8·27	986395	·53	405308	8·81	594692	44
17	392199	8·26	986363	·54	405836	8·80	594164	43
18	392695	8·25	986331	·54	406364	8·79	593636	42
19	393191	8·24	986299	·54	406892	8·78	593108	41
20	393685	8·23	986266	·54	407419	8·77	592581	40
21	9·394179	8·22	9·986234	·54	9·407945	8·76	10·592055	39
22	394673	8·21	986202	·54	408471	8·75	591529	38
23	395166	8·20	986169	·54	408997	8·74	591003	37
24	395658	8·19	986137	·54	409521	8·74	590479	36
25	396150	8·18	986104	·54	410045	8·73	589955	35
26	396641	8·17	986072	·54	410569	8·72	589431	34
27	397132	8·17	986039	·54	411092	8·71	588908	33
28	397621	8·16	986007	·54	411615	8·70	588385	32
29	398111	8·15	985974	·54	412137	8·69	587863	31
30	398600	8·14	985942	·54	412658	8·68	587342	30
31	9·399088	8·13	9·985909	·55	9·413179	8·67	10·586821	29
32	399575	8·12	985876	·55	413699	8·66	586301	28
33	400062	8·11	985843	·55	414219	8·65	585781	27
34	400549	8·10	985811	·55	414738	8·64	585262	26
35	401035	8·09	985778	·55	415257	8·64	584743	25
36	401520	8·08	985745	·55	415775	8·63	584225	24
37	402005	8·07	985712	·55	416293	8·62	583707	23
38	402489	8·06	985679	·55	416810	8·61	583190	22
39	402972	8·05	985646	·55	417326	8·60	582674	21
40	403455	8·04	985613	·55	417842	8·59	582158	20
41	9·403938	8·03	9·985580	·55	9·418358	8·58	10·581642	19
42	404420	8·02	985547	·55	418873	8·57	581127	18
43	404901	8·01	985514	·55	419387	8·56	580613	17
44	405382	8·00	985480	·55	419901	8·55	580099	16
45	405862	7·99	985447	·55	420415	8·55	579585	15
46	406341	7·98	985414	·56	420927	8·54	579073	14
47	406820	7·97	985380	·56	421440	8·53	578560	13
48	407299	7·96	985347	·56	421952	8·52	578048	12
49	407777	7·95	985314	·56	422463	8·51	577537	11
50	408254	7·94	985280	·56	422974	8·50	577026	10
51	9 408731	7·94	9·985247	·56	9·423484	8·49	10·576516	9
52	409207	7·93	985213	·56	423993	8·48	576007	8
53	409682	7·92	985180	·56	424503	8·48	575497	7
54	410157	7·91	985146	·56	425011	8·47	574989	6
55	410632	7·90	985113	·56	425519	8·46	574481	5
56	411106	7·89	985079	·56	426027	8·45	573973	4
57	411579	7·88	985045	·56	426534	8·44	573466	3
58	412052	7·87	985011	·56	427041	8·43	572959	2
59	412524	7·86	984978	·56	427547	8·43	572453	1
60	412996	7·85	984944	56	428052	8·42	571948	0
	Cosine	D.	Sine	75°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9·412996	7·85	9·984944	·57	9·428052	8·42	10·571948	60
1	413467	7·84	984910	·57	428557	8·41	571443	59
2	413938	7·83	984876	·57	429062	8·40	570938	58
3	414408	7·83	984842	·57	429566	8·39	570434	57
4	414878	7·82	984808	·57	430070	8·38	569930	56
5	415347	7·81	984774	·57	430573	8·38	569427	55
6	415815	7·80	984740	·57	431075	8·37	568925	54
7	416283	7·79	984706	·57	431577	8·36	568423	53
8	416751	7·78	984672	·57	432079	8·35	567921	52
9	417217	7·77	984637	·57	432580	8·34	567420	51
10	417684	7·76	984603	·57	433080	8·33	566920	50
11	9·418150	7·75	9·984569	·57	9·433580	8·32	10·566420	49
12	418615	7·74	984535	·57	434080	8·32	565920	48
13	419079	7·73	984500	·57	434579	8·31	565421	47
14	419544	7·73	984466	·57	435078	8·30	564922	46
15	420007	7·72	984432	·58	435576	8·29	564424	45
16	420470	7·71	984397	·58	436073	8·28	563927	44
17	420933	7·70	984363	·58	436570	8·28	563430	43
18	421395	7·69	984328	·58	437067	8·27	562933	42
19	421857	7·68	984294	·58	437563	8·26	562437	41
20	422318	7·67	984259	·58	438059	8·25	561941	40
21	9·422778	7·67	9·984224	·58	9·438554	8·24	10·561446	39
22	423238	7·66	984190	·58	439048	8·23	560952	38
23	423697	7·65	984155	·58	439543	8·23	560457	37
24	424156	7·64	984120	·58	440036	8·22	559964	36
25	424615	7·63	984085	·58	440529	8·21	559471	35
26	425073	7·62	984050	·58	441022	8·20	558978	34
27	425530	7·61	984015	·58	441514	8·19	558486	33
28	425987	7·60	983981	·58	442006	8·19	557994	32
29	426443	7·60	983946	·58	442497	8·18	557503	31
30	426899	7·59	983911	·58	442988	8·17	557012	30
31	9·427354	7·58	9·983875	·58	9·443479	8·16	10·556521	29
32	427809	7·57	983840	·59	443968	8·16	556032	28
33	428263	7·56	983805	·59	444458	8·15	555542	27
34	428717	7·55	983770	·59	444947	8·14	555053	26
35	429170	7·54	983735	·59	445435	8·13	554565	25
36	429623	7·53	983700	·59	445923	8·12	554077	24
37	430075	7·52	983664	·59	446411	8·12	553589	23
38	430527	7·52	983629	·59	446898	8·11	553102	22
39	430978	7·51	983594	·59	447384	8·10	552616	21
40	431429	7·50	983558	·59	447870	8·09	552130	20
41	9·431879	7·49	9·983523	·59	9·448356	8·09	10·551644	19
42	432329	7·49	983487	·59	448841	8·08	551159	18
43	432778	7·48	983452	·59	449326	8·07	550674	17
44	433226	7·47	983416	·59	449810	8·06	550190	16
45	433675	7·46	983381	·59	450294	8·06	549706	15
46	434122	7·45	983345	·59	450777	8·05	549223	14
47	434569	7·44	983309	·59	451260	8·04	548740	13
48	435016	7·44	983273	·60	451743	8·03	548257	12
49	435462	7·43	983238	·60	452225	8·02	547775	11
50	435908	7·42	983202	·60	452706	8·02	547294	10
51	9·436353	7·41	9·983166	·60	9·453187	8·01	10·546813	9
52	436798	7·40	983130	·60	453668	8·00	546332	8
53	437242	7·40	983094	·60	454148	7·99	545852	7
54	437686	7·39	983058	·60	454628	7·99	545372	6
55	438129	7·38	983022	·60	455107	7·98	544893	5
56	438572	7·37	982986	·60	455586	7·97	544414	4
57	439014	7·36	982950	·60	456064	7·96	543936	3
58	439456	7·36	982914	·60	456542	7·96	543458	2
59	439897	7·35	982878	·60	457019	7·95	542981	1
60	440338	7·34	982842	·60	457496	7·94	542504	0
	Cosine	D.	Sine	74°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·440338	7·34	9·982842	·60	9·457496	7·94	10·542504	60
1	440778	7·33	982805	·60	457973	7·93	542027	59
2	441218	7·32	982769	·61	458449	7·93	541551	58
3	441658	7·31	982733	·61	458925	7·92	541075	57
4	442096	7·31	982696	·61	459400	7·91	540600	56
5	442535	7·30	982660	·61	459875	7·90	540125	55
6	442973	7·29	982624	·61	460349	7·90	539651	54
7	443410	7·28	982587	·61	460823	7·89	539177	53
8	443847	7·27	982551	·61	461297	7·88	538703	52
9	444284	7·27	982514	·61	461770	7·88	538230	51
10	444720	7·26	982477	·61	462242	7·87	537758	50
11	9·445155	7·25	9·982441	·61	9·462714	7·86	10·537286	49
12	445590	7·24	982404	·61	463186	7·85	536814	48
13	446025	7·23	982367	·61	463658	7·85	536342	47
14	446459	7·23	982331	·61	464129	7·84	535871	46
15	446893	7·22	982294	·61	464599	7·83	535401	45
16	447326	7·21	982257	·61	465069	7·83	534931	44
17	447759	7·20	982220	·62	465539	7·82	534461	43
18	448191	7·20	982183	·62	466008	7·81	533992	42
19	448623	7·19	982146	·62	466476	7·80	533524	41
20	449054	7·18	982109	·62	466945	7·80	533055	40
21	9·449485	7·17	9·982072	·62	9·467413	7·79	10·532587	39
22	449915	7·16	982035	·62	467880	7·78	532120	38
23	450345	7·16	981998	·62	468347	7·78	531653	37
24	450775	7·15	981961	·62	468814	7·77	531186	36
25	451204	7·14	981924	·62	469280	7·76	530720	35
26	451632	7·13	981886	·62	469746	7·75	530254	34
27	452060	7·13	981849	·62	470211	7·75	529789	33
28	452488	7·12	981812	·62	470676	7·74	529324	32
29	452915	7·11	981774	·62	471141	7·73	528859	31
30	453342	7·10	981737	·62	471605	7·73	528395	30
31	9·453768	7·10	9·981699	·63	9·472068	7·72	10·527932	29
32	454194	7·09	981662	·63	472532	7·71	527468	28
33	454619	7·08	981625	·63	472995	7·71	527005	27
34	455044	7·07	981587	·63	473457	7·70	526543	26
35	455469	7·07	981549	·63	473919	7·69	526081	25
36	455893	7·06	981512	·63	474381	7·69	525619	24
37	456316	7·05	981474	·63	474842	7·68	525158	23
38	456739	7·04	981436	·63	475303	7·67	524697	22
39	457162	7·04	981399	·63	475763	7·67	524237	21
40	457584	7·03	981361	·63	476223	7·66	523777	20
41	9·458006	7·02	9·981323	·63	9·476683	7·65	10·523317	19
42	458427	7·01	981285	·63	477142	7·65	522858	18
43	458848	7·01	981247	·63	477601	7·64	522399	17
44	459268	7·00	981209	·63	478059	7·63	521941	16
45	459688	6·99	981171	·63	478517	7·63	521483	15
46	460108	6·98	981133	·64	478975	7·62	521025	14
47	460527	6·98	981095	·64	479432	7·61	520568	13
48	460946	6·97	981057	·64	479889	7·61	520111	12
49	461364	6·96	981019	·64	480345	7·60	519655	11
50	461782	6·95	980981	·64	480801	7·59	519199	10
51	9·462199	6·95	9·980942	·64	9·481257	7·59	10·518743	9
52	462616	6·94	980904	·64	481712	7·58	518288	8
53	463032	6·93	980866	·64	482167	7·57	517833	7
54	463448	6·93	980827	·64	482621	7·57	517379	6
55	463864	6·92	980789	·64	483075	7·56	516925	5
56	464279	6·91	980750	·64	483529	7·55	516471	4
57	464694	6·90	980712	·64	483982	7·55	516018	3
58	465108	6·90	980673	·64	484435	7·54	515565	2
59	465522	6·89	980635	·64	484887	7·53	515113	1
60	465935	6·88	980596	·64	485339	7·53	514661	0
	Cosine	D.	Sine	73°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·465935	6·88	9·980596	·64	9·485339	7·55	10·514661	60
1	466348	6·88	980558	·64	485791	7·52	514209	59
2	466761	6·87	980519	65	486242	7·51	513758	58
3	467173	6·86	980480	·65	486693	7·51	513307	57
4	467585	6·85	980442	·65	487143	7·50	512857	56
5	467996	6·85	980403	·65	487593	7·49	512407	55
6	468407	6·84	980364	·65	488043	7·49	511957	54
7	468817	6·83	980325	·65	488492	7·48	511508	53
8	469227	6·83	980286	·65	488941	7·47	511059	52
9	469637	6·82	980247	·65	489390	7·47	510610	51
10	470046	6·81	980208	·65	489838	7·46	510162	50
11	9·470455	6·80	9·980169	·65	9·490286	7·46	10·509714	49
12	470863	6·80	980130	·65	490733	7·45	509267	48
13	471271	6·79	980091	·65	491180	7·44	508820	47
14	471679	6·78	980052	·65	491627	7·44	508373	46
15	472086	6·78	980012	·65	492073	7·43	507927	45
16	472492	6·77	979973	·65	492519	7·43	507481	44
17	472898	6·76	979934	·66	492965	7·42	507035	43
18	473304	6·76	979895	·66	493410	7·41	506590	42
19	473710	6·75	979855	·66	493854	7·40	506146	41
20	474115	6·74	979816	·66	494299	7·40	505701	40
21	9·474519	6·74	9·979776	·66	9·494743	7·40	10·505257	39
22	474923	6·73	979737	·66	495186	7·39	504814	38
23	475327	6·72	979697	·66	495630	7·38	504370	37
24	475730	6·72	979658	·66	496073	7·37	503927	36
25	476133	6·71	979618	·66	496515	7·37	503485	35
26	476536	6·70	979579	·66	496957	7·36	503043	34
27	476938	6·69	979539	·66	497399	7·36	502601	33
28	477340	6·69	979499	·66	497841	7·35	502159	32
29	477741	6·68	979459	·66	498282	7·34	501718	31
30	478142	6·67	979420	·66	498722	7·34	501278	30
31	9·478542	6·67	9·979380	·66	9·499163	7·33	10·500837	29
32	478942	6·66	979340	·66	499603	7·33	500397	28
33	479342	6·65	979300	·67	500042	7·32	499958	27
34	479741	6·65	979260	·67	500481	7·31	499519	26
35	480140	6·64	979220	·67	500920	7·31	499080	25
36	480539	6·63	979180	·67	501359	7·30	498641	24
37	480937	6·63	979140	·67	501797	7·30	498203	23
38	481334	6·62	979100	·67	502235	7·29	497765	22
39	481731	6·61	979059	·67	502672	7·28	497328	21
40	482128	6·61	979019	·67	503109	7·28	496891	20
41	9·482525	6·60	9·978979	·67	9·503546	7·27	10·496454	19
42	482921	6·59	978939	·67	503982	7·27	496018	18
43	483316	6·59	978898	·67	504418	7·26	495582	17
44	483712	6·58	978858	·67	504854	7·25	495146	16
45	484107	6·57	978817	·67	505289	7·25	494711	15
46	484501	6·57	978777	·67	505724	7·24	494276	14
47	484895	6·56	978736	·67	506159	7·24	493841	13
48	485289	6·55	978696	·68	506593	7·23	493407	12
49	485682	6·55	978655	·68	507027	7·22	492973	11
50	486075	6·54	978615	·68	507460	7·22	492540	10
51	9·486467	6·53	9·978574	·68	9·507893	7·21	10·492107	9
52	486860	6·53	978533	·68	508326	7·21	491674	8
53	487251	6·52	978493	·68	508759	7·20	491241	7
54	487643	6·51	978452	·68	509191	7·19	490809	6
55	488034	6·51	978411	·68	509622	7·19	490378	5
56	488424	6·50	978370	·68	510054	7·18	489946	4
57	488814	6·50	978329	·68	510485	7·18	489515	3
58	489204	6·49	978288	·68	510916	7·17	489084	2
59	489593	6·48	978247	·68	511346	7·16	488654	1
60	489982	6·48	978206	·68	511776	7·16	488224	0
	Cosine	D.	Sine	72°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·489982	6·48	9·978206	·68	9·511776	7·16	10·488224	60
1	490371	6·48	978165	·68	512206	7·16	487794	59
2	490759	6·47	978124	·68	512635	7·15	487365	58
3	491147	6·46	978083	·69	513064	7·14	486936	57
4	491535	6·46	978042	·69	513493	7·14	486507	56
5	491922	6·45	978001	·69	513921	7·13	486079	55
6	492308	6·44	977959	·69	514349	7·13	485651	54
7	492695	6·44	977918	·69	514777	7·12	485223	53
8	493081	6·43	977877	·69	515204	7·12	484796	52
9	493466	6·42	977835	·69	515631	7·11	484369	51
10	493851	6·42	977794	·69	516057	7·10	483943	50
11	9·494236	6·41	9·977752	·69	9·516484	7·10	10·483516	49
12	494621	6·41	977711	·69	516910	7·09	483090	48
13	495005	6·40	977669	·69	517335	7·09	482665	47
14	495388	6·39	977628	·69	517761	7·08	482239	46
15	495772	6·39	977586	·69	518185	7·08	481815	45
16	496154	6·38	977544	·70	518610	7·07	481390	44
17	496537	6·37	977503	·70	519034	7·06	480966	43
18	496919	6·37	977461	·70	519458	7·06	480542	42
19	497301	6·36	977419	·70	519882	7·05	480118	41
20	497682	6·36	977377	·70	520305	7·05	479695	40
21	9·498064	6·35	9·977335	·70	9·520728	7·04	10·479272	39
22	498444	6·34	977293	·70	521151	7·03	478849	38
23	498825	6·34	977251	·70	521573	7·03	478427	37
24	499204	6·33	977209	·70	521995	7·03	478005	36
25	499584	6·32	977167	·70	522417	7·02	477583	35
26	499963	6·32	977125	·70	522838	7·02	477162	34
27	500342	6·31	977083	·70	523259	7·01	476741	33
28	500721	6·31	977041	·70	523680	7·01	476320	32
29	501099	6·30	976999	·70	524100	7·00	475900	31
30	501476	6·29	976957	·70	524520	6·99	475480	30
31	9·501854	6·29	9·976914	·70	9·524939	6·99	10·475061	29
32	502231	6·28	976872	·71	525359	6·98	474641	28
33	502607	6·28	976830	·71	525778	6·98	474222	27
34	502984	6·27	976787	·71	526197	6·97	473803	26
35	503360	6·26	976745	·71	526615	6·97	473385	25
36	503735	6·26	976702	·71	527033	6·96	472967	24
37	504110	6·25	976660	·71	527451	6·96	472549	23
38	504485	6·25	976617	·71	527868	6·95	472132	22
39	504860	6·24	976574	·71	528285	6·95	471715	21
40	505234	6·23	976532	·71	528702	6·94	471298	20
41	9·505608	6·23	9·976489	·71	9·529119	6·93	10·470881	19
42	505981	6·22	976446	·71	529535	6·93	470465	18
43	506354	6·22	976404	·71	529950	6·93	470050	17
44	506727	6·21	976361	·71	530366	6·92	469634	16
45	507099	6·20	676318	·71	530781	6·91	469219	15
46	507471	6·20	976275	·71	531196	6·91	468804	14
47	507843	6·19	976232	·72	531611	6·90	468389	13
48	508214	6·19	976189	·72	532025	6·90	467975	12
49	508585	6·18	976146	·72	532439	6·89	467561	11
50	508956	6·18	976103	·72	532853	6·89	467147	10
51	9·509326	6·17	9·976060	·72	9·533266	6·88	10·466734	9
52	509696	6·16	976017	·72	533679	6·88	466321	8
53	510065	6·16	975974	·72	534092	6·87	465908	7
54	510434	6·15	975930	·72	534504	6·87	465496	6
55	510803	6·15	975887	·72	534916	6·86	465084	5
56	511172	6·14	975844	·72	535328	6·86	464672	4
57	511540	6·13	975800	·72	535739	6·85	464261	3
58	511907	6·13	975757	·72	536150	6·85	463850	2
59	512275	6·12	975714	·72	536561	6·84	463439	1
60	512642	6·12	975670	·72	536972	6·84	463028	0
	Cosine	D.	Sine	71°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·512642	6·12	9·975670	·73	9·536972	6·84	10·463028	60
1	513009	6·11	975627	·73	537382	6·83	462618	59
2	513375	6·11	975583	·73	537792	6·83	462208	58
3	513741	6·10	975539	·73	538202	6·82	461798	57
4	514107	6·09	975496	·73	538611	6·82	461389	56
5	514472	6·09	975452	·73	539020	6·81	460980	55
6	514837	6·08	975408	·73	539429	6·81	460571	54
7	515202	6·08	975365	·73	539837	6·80	460163	53
8	515566	6·07	975321	·73	540245	6·80	459755	52
9	515930	6·07	975277	·73	540653	6·79	459347	51
10	516294	6·06	975233	·73	541061	6·79	458939	50
11	9·516657	6·05	9·975189	·73	9·541468	6·78	10·458532	49
12	517020	6·05	975145	·73	541875	6·78	458125	48
13	517382	6·04	975101	·73	542281	6·77	457719	47
14	517745	6·04	975057	·73	542688	6·77	457312	46
15	518107	6·03	975013	·73	543094	6·76	456906	45
16	518468	6·03	974969	·74	543499	6·76	456501	44
17	518829	6·02	974925	·74	543905	6·75	456095	43
18	519190	6·01	974880	·74	544310	6·75	455690	42
19	519551	6·01	974836	·74	544715	6·74	455285	41
20	519911	6·00	974792	·74	545119	6·74	454881	40
21	9·520271	6·00	9·974748	·74	9·545524	6·73	10·454476	39
22	520631	5·99	974703	·74	545928	6·73	454072	38
23	520990	5·99	974659	·74	546331	6·72	453669	37
24	521349	5·98	974614	·74	546735	6·72	453265	36
25	521707	5·98	974570	·74	547138	6·71	452862	35
26	522066	5·97	974525	·74	547540	6·71	452460	34
27	522424	5·96	974481	·74	547943	6·70	452057	33
28	522781	5·96	974436	·74	548345	6·70	451655	32
29	523138	5·95	974391	·74	548747	6·69	451253	31
30	523495	5·95	974347	·75	549149	6·69	450851	30
31	9·523852	5·94	9·974302	·75	9·549550	6·68	10·450450	29
32	524208	5·94	974257	·75	549951	6·68	450049	28
33	524564	5·93	974212	·75	550352	6·67	449648	27
34	524920	5·93	974167	·75	550752	6·67	449248	26
35	525275	5·92	974122	·75	551152	6·66	448848	25
36	525630	5·91	974077	·75	551552	6·66	448448	24
37	525984	5·91	974032	·75	551952	6·65	448048	23
38	526339	5·90	973987	·75	552351	6·65	447649	22
39	526693	5·90	973942	·75	552750	6·65	447250	21
40	527046	5·89	973897	·75	553149	6·64	446851	20
41	9·527400	5·89	9·973852	·75	9·553548	6·64	10·446452	19
42	527753	5·88	973807	·75	553946	6·63	446054	18
43	528105	5·88	973761	·75	554344	6·63	445656	17
44	528458	5·87	973716	·76	554741	6·62	445259	16
45	528810	5·87	973671	·76	555139	6·62	444861	15
46	529161	5·86	973625	·76	555536	6·61	444464	14
47	529513	5·86	973580	·76	555933	6·61	444067	13
48	529864	5·85	973535	·76	556329	6·60	443671	12
49	530215	5·85	973489	·76	556725	6·60	443275	11
50	530565	5·84	973444	·76	557121	6·59	442879	10
51	9·530915	5·84	9·973398	·76	9·557517	6·59	10·442483	9
52	531265	5·83	973352	·76	557913	6·59	442087	8
53	531614	5·82	973307	·76	558308	6·58	441692	7
54	531963	5·82	973261	·76	558702	6·58	441298	6
55	532312	5·81	973215	·76	559097	6·57	440903	5
56	532661	5·81	973169	·76	559491	6·57	440509	4
57	533009	5·80	973124	·76	559885	6·56	440115	3
58	533357	5·80	973078	·76	560279	6·56	439721	2
59	533704	5·79	973032	·77	560673	6·55	439327	1
60	534052	5·78	972986	·77	561066	6·55	438934	0
	Cosine	D.	Sine	70°	Cotang.	D	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·534052	5·78	9·972986	·77	9·561066	6·55	10·438934	60
1	534399	5·77	972940	·77	561459	6·54	438541	59
2	534745	5·77	972894	·77	561851	6·54	438149	58
3	535092	5·77	972848	·77	562244	6·53	437756	57
4	535438	5·76	972802	·77	562636	6·53	437364	56
5	535783	5·76	972755	·77	563028	6·53	436972	55
6	536129	5·75	972709	·77	563419	6·52	436581	54
7	536474	5·74	972663	·77	563811	6·52	436189	53
8	536818	5·74	972617	·77	564202	6·51	435798	52
9	537163	5·73	972570	·77	564592	6·51	435408	51
10	537507	5·73	972524	·77	564983	6·50	435017	50
11	9·537851	5·72	9·972478	·77	9·565373	6·50	10·434627	49
12	538194	5·72	972431	·78	565763	6·49	434237	48
13	538538	5·71	972385	·78	566153	6·49	433847	47
14	538880	5·71	972338	·78	566542	6·49	433458	46
15	539223	5·70	972291	·78	566932	6·48	433068	45
16	539565	5·70	972245	·78	567320	6·48	432680	44
17	539907	5·69	972198	·78	567709	6·47	432291	43
18	540249	5·69	972151	·78	568098	6·47	431902	42
19	540590	5·68	972105	·78	568486	6·46	431514	41
20	540931	5·68	972058	·78	568873	6·46	431127	40
21	9·541272	5·67	9·972011	·78	9·569261	6·45	10·430739	39
22	541613	5·67	971964	·78	569648	6·45	430352	38
23	541953	5·66	971917	·78	570035	6·45	429965	37
24	542293	5·66	971870	·78	570422	6·44	429578	36
25	542632	5·65	971823	·78	570809	6·44	429191	35
26	542971	5·65	971776	·78	571195	6·43	428805	34
27	543310	5·64	971729	·79	571581	6·43	428419	33
28	543649	5·64	971682	·79	571967	6·42	428033	32
29	543987	5·63	971635	·79	572352	6·42	427648	31
30	544325	5·63	971588	·79	572738	6·42	427262	30
31	9·544663	5·62	9·971540	·79	9·573123	6·41	10·426877	29
32	545000	5·62	971493	·79	573507	6·41	426493	28
33	545338	5·61	971446	·79	573892	6·40	426108	27
34	545674	5·61	971398	·79	574276	6·40	425724	26
35	546011	5·60	971351	·79	574660	6·39	425340	25
36	546347	5·60	971303	·79	575044	6·39	424956	24
37	546683	5·59	971256	·79	575427	6·39	424573	23
38	547019	5·59	971208	·79	575810	6·38	424190	22
39	547354	5·58	971161	·79	576193	6·38	423807	21
40	547689	5·58	971113	·79	576576	6·37	423424	20
41	9·548024	5·57	9·971066	·80	9·576958	6·37	10·423041	19
42	548359	5·57	971018	·80	577341	6·36	422659	18
43	548693	5·56	970970	·80	577723	6·36	422277	17
44	549027	5·56	970922	·80	578104	6·36	421896	16
45	549360	5·55	970874	·80	578486	6·35	421514	15
46	549693	5·55	970827	·80	578867	6·35	421133	14
47	550026	5·54	970779	·80	579248	6·34	420752	13
48	550359	5·54	970731	·80	579629	6·34	420371	12
49	550692	5·53	970683	·80	580009	6·34	419991	11
50	551024	5·53	970635	·80	580389	6·33	419611	10
51	9·551356	5·52	9·970586	·80	9·580769	6·33	10·419231	9
52	551687	5·52	970538	·80	581149	6·32	418851	8
53	552018	5·52	970490	·80	581528	6·32	418472	7
54	552349	5·51	970442	·80	581907	6·32	418093	6
55	552680	5·51	970394	·80	582286	6·31	417714	5
56	553010	5·50	970345	·81	582665	6·31	417335	4
57	553341	5·50	970297	·81	583043	6·30	416957	3
58	553670	5·49	970249	·81	583422	6·30	416578	2
59	554000	5·49	970200	·81	583800	6·29	416200	1
60	554329	5·48	970152	·81	584177	6·29	415823	0
	Cosine	D.	Sine	69°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·554329	5·48	9·970152	·81	9·584177	6·29	10·415823	60
1	554658	5·48	970103	·81	584555	6·29	415445	59
2	554987	5·47	970055	·81	584932	6·28	415068	58
3	555315	5·47	970006	·81	585309	6·28	414691	57
4	555643	5·46	969957	·81	585686	6·27	414314	56
5	555971	5·46	969909	·81	586062	6·27	413938	55
6	556299	5·45	969860	·81	586439	6·27	413561	54
7	556626	5·45	969811	·81	586815	6·26	413185	53
8	556953	5·44	969762	·81	587190	6·26	412810	52
9	557280	5·44	969714	·81	587566	6·25	412434	51
10	557606	5·43	969665	·81	587941	6·25	412059	50
11	9·557932	5·43	9·969616	·82	9·588316	6·25	10·411684	49
12	558258	5·43	969567	·82	588691	6·24	411309	48
13	558583	5·42	969518	·82	589066	6·24	410934	47
14	558909	5·42	969469	·82	589440	6·23	410560	46
15	559234	5·41	969420	·82	589814	6·23	410186	45
16	559558	5·41	969370	·82	590188	6·23	409812	44
17	559883	5·40	969321	·82	590562	6·22	409438	43
18	560207	5·40	969272	·82	590935	6·22	409065	42
19	560531	5·39	969223	·82	591308	6·22	408692	41
20	560855	5·39	969173	·82	591681	6·21	408319	40
21	9·561178	5·38	9·969124	·82	9·592054	6·21	10·407946	39
22	561501	5·38	969075	·82	592426	6·20	407574	38
23	561824	5·37	969025	·82	592798	6·20	407202	37
24	562146	5·37	968976	·82	593170	6·19	406829	36
25	562468	5·36	968926	·83	593542	6·19	406458	35
26	562790	5·36	968877	·83	593914	6·18	406086	34
27	563112	5·36	968827	·83	594285	6·18	405715	33
28	563433	5·35	968777	·83	594656	6·18	405344	32
29	563755	5·35	968728	·83	595027	6·17	404973	31
30	564075	5·34	968678	·83	595398	6·17	404602	30
31	9·564396	5·34	9·968628	·83	9·595768	6·17	10·404232	29
32	564716	5·33	968578	·83	596138	6·16	403862	28
33	565036	5·33	968528	·83	596508	6·16	403492	27
34	565356	5·32	968479	·83	596878	6·16	403122	26
35	565676	5·32	968429	·83	597247	6·15	402753	25
36	565995	5·31	968379	·83	597616	6·15	402384	24
37	566314	5·31	968329	·83	597985	6·15	402015	23
38	566632	5·31	968278	·83	598354	6·14	401646	22
39	566951	5·30	968228	·84	598722	6·14	401278	21
40	567269	5·30	968178	·84	599091	6·13	400909	20
41	9·567587	5·29	9·968128	·84	9·599459	6·13	10·400541	19
42	567904	5·29	968078	·84	599827	6·13	400173	18
43	568222	5·28	968027	·84	600194	6·12	399806	17
44	568539	5·28	967977	·84	600562	6·12	399438	16
45	568856	5·28	967927	·84	600929	6·11	399071	15
46	569172	5·27	967876	·84	601296	6·11	398704	14
47	569488	5·27	967826	·84	601662	6·11	398338	13
48	569804	5·26	967775	·84	602029	6·10	397971	12
49	570120	5·26	967725	·84	602395	6·10	397605	11
50	570435	5·25	967674	·84	602761	6·10	397239	10
51	9·570751	5·25	9·967624	·84	9·603127	6·09	10·396873	9
52	571066	5·24	967573	·84	603493	6·09	396507	8
53	571380	5·24	967522	·85	603858	6·09	396142	7
54	571695	5·23	967471	·85	604223	6·08	395777	6
55	572009	5·23	967421	·85	604588	6·08	395412	5
56	572323	5·23	967370	·85	604953	6·07	395047	4
57	572636	5·22	967319	·85	605317	6·07	394683	3
58	572950	5·22	967268	·85	605682	6·07	394318	2
59	573263	5·21	967217	·85	606046	6·06	393954	1
60	573575	5·21	967166	·85	606410	6·06	393590	0
	Cosine	D.	Sine	68°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·573575	5·21	9·967166	·85	9·606410	6·06	10·393590	60
1	573888	5·20	967115	·85	606773	6·06	393227	59
2	574200	5·20	967064	·85	607137	6·05	392863	58
3	574512	5·19	967013	·85	607500	6·05	392500	57
4	574824	5·19	966961	·85	607863	6·04	392137	56
5	575136	5·19	966910	·85	608225	6·04	391775	55
6	575447	5·18	966859	·85	608588	6·04	391412	54
7	575758	5·18	966808	·85	608950	6·03	391050	53
8	576069	5·17	966756	·86	609312	6·03	390688	52
9	576379	5·17	966705	·86	609674	6·03	390326	51
10	576689	5·16	966653	·86	610036	6·02	389964	50
11	9·576999	5·16	9·966602	·86	9·610397	6·02	10·389603	49
12	577309	5·16	966550	·86	610759	6·02	389241	48
13	577618	5·15	966499	·86	611120	6·01	388880	47
14	577927	5·15	966447	·86	611480	6·01	388520	46
15	578236	5·14	966395	·86	611841	6·01	388159	45
16	578545	5·14	966344	·86	612201	6·00	387799	44
17	578853	5·13	966292	·86	612561	6·00	387439	43
18	579162	5·13	966240	·86	612921	6·00	387079	42
19	579470	5·13	966188	·86	613281	5·99	386719	41
20	579777	5·12	966136	·86	613641	5·99	386359	40
21	9·580085	5·12	9·966085	·87	9·614000	5·98	10·386000	39
22	580392	5·11	966033	·87	614359	5·98	385641	38
23	580699	5·11	965981	·87	614718	5·98	385282	37
24	581005	5·11	965928	·87	615077	5·97	384923	36
25	581312	5·10	965876	·87	615435	5·97	384565	35
26	581618	5·10	965824	·87	615793	5·97	384207	34
27	581924	5·09	965772	·87	616151	5·96	383849	33
28	582229	5·09	965720	·87	616509	5·96	383491	32
29	582535	5·09	965668	·87	616867	5·96	383133	31
30	582840	5·08	965615	·87	617224	5·95	382776	30
31	9·583145	5·08	9·965563	·87	9·617582	5·95	10·382418	29
32	583449	5·07	965511	·87	617939	5·95	382061	28
33	583754	5·07	965458	·87	618295	5·94	381705	27
34	584058	5·06	965406	·87	618652	5·94	381348	26
35	584361	5·06	965353	·88	619008	5·94	380992	25
36	584665	5·06	965301	·88	619364	5·93	380636	24
37	584968	5·05	965248	·88	619721	5·93	380279	23
38	585272	5·05	965195	·88	620076	5·93	379924	22
39	585574	5·04	965143	·88	620432	5·92	379568	21
40	585877	5·04	965090	·88	620787	5·92	379213	20
41	9·586179	5·03	9·965037	·88	9·621142	5·92	10·378858	19
42	586482	5·03	964984	·88	621497	5·91	378503	18
43	586783	5·03	964931	·88	621852	5·91	378148	17
44	587085	5·02	964879	·88	622207	5·90	377793	16
45	587386	5·02	964826	·88	622561	5·90	377439	15
46	587688	5·01	964773	·88	622915	5·90	377085	14
47	587989	5·01	964719	·88	623269	5·89	376731	13
48	588289	5·01	964666	·89	623623	5·89	376377	12
49	588590	5·00	964613	·89	623976	5·89	376024	11
50	588890	5·00	964560	·89	624330	5·88	375670	10
51	9·589190	4·99	9·964507	·89	9·624683	5·88	10·375317	9
52	589489	4·99	964454	·89	625036	5·88	374964	8
53	589789	4·99	964400	·89	625388	5·87	374612	7
54	590088	4·98	964347	·89	625741	5·87	374259	6
55	590387	4·98	964294	·89	626093	5·87	373907	5
56	590686	4·97	964240	·89	626445	5·86	373555	4
57	590984	4·97	964187	·89	626797	5·86	373203	3
58	591282	4·97	964133	·89	627149	5·86	372851	2
59	591580	4·96	964080	·89	627501	5·85	372499	1
60	591878	4·96	964026	·89	627852	5·85	372148	0
	Cosine	D.	Sine	67°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·591878	4·96	9·964026	·89	9·627852	5·85	10·372148	60
1	592176	4·95	963972	·89	628203	5·85	371797	59
2	592473	4·95	963919	·89	628554	5·85	371446	58
3	592770	4·95	963865	·90	628905	5·84	371095	57
4	593067	4·94	963811	·90	629255	5·84	370745	56
5	593363	4·94	963757	·90	629606	5·83	370394	55
6	593659	4·93	963704	·90	629956	5·83	370044	54
7	593955	4·93	963650	·90	630306	5·83	369694	53
8	594251	4·93	963596	·90	630656	5·83	369344	52
9	594547	4·92	963542	·90	631005	5·82	368995	51
10	594842	4·92	963488	·90	631355	5·82	368645	50
11	9·595137	4·91	9·963434	·90	9·631704	5·82	10·368296	49
12	595432	4·91	963379	·90	632053	5·81	367947	48
13	595727	4·91	963325	·90	632401	5·81	367599	47
14	596021	4·90	963271	·90	632750	5·81	367250	46
15	596315	4·90	963217	·90	633098	5·80	366902	45
16	596609	4·89	963163	·90	633447	5·80	366553	44
17	596903	4·89	963108	·91	633795	5·80	366205	43
18	597196	4·89	963054	·91	634143	5·79	365857	42
19	597490	4·88	962999	·91	634490	5·79	365510	41
20	597783	4·88	962945	·91	634838	5·79	365162	40
21	9·598075	4·87	9·962890	·91	9·635185	5·78	10·364815	39
22	598368	4·87	962836	·91	635532	5·78	364468	38
23	598660	4·87	962781	·91	635879	5·78	364121	37
24	598952	4·86	962727	·91	636226	5·77	363774	36
25	599244	4·86	962672	·91	636572	5·77	363428	35
26	599536	4·85	962617	·91	636919	5·77	363081	34
27	599827	4·85	962562	·91	637265	5·77	362735	33
28	600118	4·85	962508	·91	637611	5·76	362389	32
29	600409	4·84	962453	·91	637956	5·76	362044	31
30	600700	4·84	962398	·92	638302	5·76	361698	30
31	9·600990	4·84	9·962343	·92	9·638647	5·75	10·361353	29
32	601280	4·83	962288	·92	638992	5·75	361008	28
33	601570	4·83	962233	·92	639337	5·75	360663	27
34	601860	4·82	962178	·92	639682	5·74	360318	26
35	602150	4·82	962123	·92	640027	5·74	359973	25
36	602439	4·82	962067	·92	640371	5·74	359629	24
37	602728	4·81	962012	·92	640716	5·73	359284	23
38	603017	4·81	961957	·92	641060	5·73	358940	22
39	603305	4·81	961902	·92	641404	5·73	358596	21
40	603594	4·80	961846	·92	641747	5·72	358253	20
41	9·603882	4·80	9·961791	·92	9·642091	5·72	10·357909	19
42	604170	4·79	961735	·92	642434	5·72	357566	18
43	604457	4·79	961680	·92	642777	5·72	357223	17
44	604745	4·79	961624	·93	643120	5·71	356880	16
45	605032	4·78	961569	·93	643463	5·71	356537	15
46	605319	4·78	961513	·93	643806	5·71	356194	14
47	605606	4·78	961458	·93	644148	5·70	355852	13
48	605892	4·77	961402	·93	644490	5·70	355510	12
49	606179	4·77	961346	·93	644832	5·70	355168	11
50	606465	4·76	961290	·93	645174	5·69	354826	10
51	9·606751	4·76	9·961235	·93	9·645516	5·69	10·354484	9
52	607036	4·76	961179	·93	645857	5·69	354143	8
53	607322	4·75	961123	·93	646199	5·69	353801	7
54	607607	4·75	961067	·93	646540	5·68	353460	6
55	607892	4·74	961011	·93	646881	5·68	353119	5
56	608177	4·74	960955	·93	647222	5·68	352778	4
57	608461	4·74	960899	·93	647562	5·67	352438	3
58	608745	4·73	960843	·94	647903	5·67	352097	2
59	609029	4·73	960786	·94	648243	5·67	351757	1
60	609313	4·73	960730	·94	648583	5·66	351417	0
	Cosine	D.	Sine	66°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·609313	4·73	9·960730	·94	9·648583	5·66	10·351417	60
1	609597	4·72	960674	·94	648923	5·66	351077	59
2	609880	4·72	960618	·94	649263	5·66	350737	58
3	610164	4·72	960561	·94	649602	5·66	350398	57
4	610447	4·71	960505	·94	649942	5·65	350058	56
5	610729	4·71	960448	·94	650281	5·65	349719	55
6	611012	4·70	960392	·94	650620	5·65	349380	54
7	611294	4·70	960335	·94	650959	5·64	349041	53
8	611576	4·70	960279	·94	651297	5·64	348703	52
9	611858	4·69	960222	·94	651636	5·64	348364	51
10	612140	4·69	960165	·94	651974	5·63	348026	50
11	9·612421	4·69	9·960109	·95	9·652312	5·63	10·347688	49
12	612702	4·68	960052	·95	652650	5·63	347350	48
13	612983	4·68	959995	·95	652988	5·63	347012	47
14	613264	4·67	959938	·95	653326	5·62	346674	46
15	613545	4·67	959882	·95	653663	5·62	346337	45
16	613825	4·67	959825	·95	654000	5·62	346000	44
17	614105	4·66	959768	·95	654337	5·61	345663	43
18	614385	4·66	959711	·95	654674	5·61	345326	42
19	614665	4·66	959654	·95	655011	5·61	344989	41
20	614944	4·65	959596	·95	655348	5·61	344652	40
21	9·615223	4·65	9·959539	·95	9·655684	5·60	10·344316	39
22	615502	4·65	959482	·95	656020	5·60	343980	38
23	615781	4·64	959425	·95	656356	5·60	343644	37
24	616060	4·64	959368	·95	656692	5·59	343308	36
25	616338	4·64	959310	·96	657028	5·59	342972	35
26	616616	4·63	959253	·96	657364	5·59	342636	34
27	616894	4·63	959195	·96	657699	5·59	342301	33
28	617172	4·62	959138	·96	658034	5·58	341966	32
29	617450	4·62	959081	·96	658369	5·58	341631	31
30	617727	4·62	959023	·96	658704	5·58	341296	30
31	9·618004	4·61	9·958965	·96	9·659039	5·58	10·340961	29
32	618281	4·61	958908	·96	659373	5·57	340627	28
33	618558	4·61	958850	·96	659708	5·57	340292	27
34	618834	4·60	958792	·96	660042	5·57	339958	26
35	619110	4·60	958734	·96	660376	5·57	339624	25
36	619386	4·60	958677	·96	660710	5·56	339290	24
37	619662	4·59	958619	·96	661043	5·56	338957	23
38	619938	4·59	958561	·96	661377	5·56	338623	22
39	620213	4·59	958503	·97	661710	5·55	338290	21
40	620488	4·58	958445	·97	662043	5·55	337957	20
41	9·620763	4·58	9·958387	·97	9·662376	5·55	10·337624	19
42	621038	4·57	958329	·97	662709	5·54	337291	18
43	621313	4·57	958271	·97	663042	5·54	336958	17
44	621587	4·57	958213	·97	663375	5·54	336625	16
45	621861	4·56	958154	·97	663707	5·54	336293	15
46	622135	4·56	958096	·97	664039	5·53	335961	14
47	622409	4·56	958038	·97	664371	5·53	335629	13
48	622682	4·55	957979	·97	664703	5·53	335297	12
49	622956	4·55	957921	·97	665035	5·53	334965	11
50	623229	4·55	957863	·97	665366	5·52	334634	10
51	9·623502	4·54	9·957804	·97	9·665697	5·52	10·334303	9
52	623774	4·54	957746	·98	666029	5·52	333971	8
53	624047	4·54	957687	·98	666360	5·51	333640	7
54	624319	4·53	957628	·98	666691	5·51	333309	6
55	624591	4·53	957570	·98	667021	5·51	332979	5
56	624863	4·53	957511	·98	667352	5·51	332648	4
57	625135	4·52	957452	·98	667682	5·50	332318	3
58	625406	4·52	957393	·98	668013	5·50	331987	2
59	625677	4·52	957335	·98	668343	5·50	331657	1
60	625948	4·51	957276	·98	668672	5·50	331328	0
	Cosine	D.	Sine	65°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·625948	4·51	9·957276	·98	9·668673	5·50	10·331327	60
1	626219	4·51	957217	·98	669002	5·49	330998	59
2	626490	4·51	957158	·98	669332	5·49	330668	58
3	626760	4·50	957099	·98	669661	5·49	330339	57
4	627030	4·50	957040	·98	669991	5·48	330009	56
5	627300	4·50	956981	·98	670320	5·48	329680	55
6	627570	4·49	956921	·99	670649	5·48	329351	54
7	627840	4·49	956862	·99	670977	5·48	329023	53
8	628109	4·49	956803	·99	671306	5·47	328694	52
9	628378	4·48	956744	·99	671634	5·47	328366	51
10	628647	4·48	956684	·99	671963	5·47	328037	50
11	9·628916	4·47	9·956625	·99	9·672291	5·47	10·327709	49
12	629185	4·47	956566	·99	672619	5·46	327381	48
13	629453	4·47	956506	·99	672947	5·46	327053	47
14	629721	4·46	956447	·99	673274	5·46	326726	46
15	629989	4·46	956387	·99	673602	5·46	326398	45
16	630257	4·46	956327	·99	673929	5·45	326071	44
17	630524	4·46	956268	·99	674257	5·45	325743	43
18	630792	4·45	956208	1·00	674584	5·45	325416	42
19	631059	4·45	956148	1·00	674910	5·44	325090	41
20	631326	4·45	956089	1·00	675237	5·44	324763	40
21	9·631593	4·44	9·956029	1·00	9·675564	5·44	10·324436	39
22	631859	4·44	955969	1·00	675890	5·44	324110	38
23	632125	4·44	955909	1·00	676216	5·43	323784	37
24	632392	4·43	955849	1·00	676543	5·43	323457	36
25	632658	4·43	955789	1·00	676869	5·43	323131	35
26	632923	4·43	955729	1·00	677194	5·43	322806	34
27	633189	4·42	955669	1·00	677520	5·42	322480	33
28	633454	4·42	955609	1·00	677846	5·42	322154	32
29	633719	4·42	955548	1·00	678171	5·42	321829	31
30	633984	4·41	955488	1·00	678496	5·42	321504	30
31	9·634249	4·41	9·955428	1·01	9·678821	5·41	10·321179	29
32	634514	4·40	955368	1·01	679146	5·41	320854	28
33	634778	4·40	955307	1·01	679471	5·41	320529	27
34	635042	4·40	955247	1·01	679795	5·41	320205	26
35	635306	4·39	955186	1·01	680120	5·40	319880	25
36	635570	4·39	955126	1·01	680444	5·40	319556	24
37	635834	4·39	955065	1·01	680768	5·40	319232	23
38	636097	4·38	955005	1·01	681092	5·40	318908	22
39	636360	4·38	954944	1·01	681416	5·39	318584	21
40	636623	4·38	954883	1·01	681740	5·39	318260	20
41	9·636886	4·37	9·954823	1·01	9·682063	5·39	10·317937	19
42	637148	4·37	954762	1·01	682387	5·39	317613	18
43	637411	4·37	954701	1·01	682710	5·38	317290	17
44	637673	4·37	954640	1·01	683033	5·38	316967	16
45	637935	4·36	954579	1·01	683356	5·38	316644	15
46	638197	4·36	954518	1·02	683679	5·38	316321	14
47	638458	4·36	954457	1·02	684001	5·37	315999	13
48	638720	4·35	954396	1·02	684324	5·37	315676	12
49	638981	4·35	954335	1·02	684646	5·37	315354	11
50	639242	4·35	954274	1·02	684968	5·37	315032	10
51	9·639503	4·34	9·954213	1·02	9·685290	5·36	10·314710	9
52	639764	4·34	954152	1·02	685612	5·36	314388	8
53	640024	4·34	954090	1·02	685934	5·36	314066	7
54	640284	4·33	954029	1·02	686255	5·36	313745	6
55	640544	4·33	953968	1·02	686577	5·35	313423	5
56	640804	4·33	953906	1·02	686898	5·35	313102	4
57	641064	4·32	953845	1·02	687219	5·35	312781	3
58	641324	4·32	953783	1·02	687540	5·35	312460	2
59	641584	4·32	953722	1·03	687861	5·34	312139	1
60	641842	4·31	953660	1·03	688182	5·34	311818	0
	Cosine	D.	Sine	64°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·641842	4·31	9·953660	1·03	9·688182	5·34	10·311818	60
1	642101	4·31	953599	1·03	688502	5·34	311498	59
2	642360	4·31	953537	1·03	688823	5·34	311177	58
3	642618	4·30	953475	1·03	689143	5·33	310857	57
4	642877	4·30	953413	1·03	689463	5·33	310537	56
5	643135	4·30	953352	1·03	689783	5·33	310217	55
6	643393	4·30	953290	1·03	690103	5·33	309897	54
7	643650	4·29	953228	1·03	690423	5·33	309577	53
8	643908	4·29	953166	1·03	690742	5·32	309258	52
9	644165	4·29	953104	1·03	691062	5·32	308938	51
10	644423	4·28	953042	1·03	691381	5·32	308619	50
11	9·644680	4·28	9·952980	1·04	9·691700	5·31	10·308300	49
12	644936	4·28	952918	1·04	692019	5·31	307981	48
13	645193	4·27	952855	1·04	692338	5·31	307662	47
14	645450	4·27	952793	1·04	692656	5·31	307344	46
15	645706	4·27	952731	1·04	692975	5·31	307025	45
16	645962	4·26	952669	1·04	693293	5·30	306707	44
17	646218	4·26	952606	1·04	693612	5·30	306388	43
18	646474	4·26	952544	1·04	693930	5·30	306070	42
19	646729	4·25	952481	1·04	694248	5·30	305752	41
20	646984	4·25	952419	1·04	694566	5·29	305434	40
21	9·647240	4·25	9·952356	1·04	9·694883	5·29	10·305117	39
22	647494	4·24	952294	1·04	695201	5·29	304799	38
23	647749	4·24	952231	1·04	695518	5·29	304482	37
24	648004	4·24	952168	1·05	695836	5·29	304164	36
25	648258	4·24	952106	1·05	696153	5·28	303847	35
26	648512	4·23	952043	1·05	696470	5·28	303530	34
27	648766	4·23	951980	1·05	696787	5·28	303213	33
28	649020	4·23	951917	1·05	697103	5·28	302897	32
29	649274	4·22	951854	1·05	697420	5·27	302580	31
30	649527	4·22	951791	1·05	697736	5·27	302264	30
31	9·649781	4·22	9·951728	1·05	9·698053	5·27	10·301947	29
32	650034	4·22	951665	1·05	698369	5·27	301631	28
33	650287	4·21	951602	1·05	698685	5·26	301315	27
34	650539	4·21	951539	1·05	699001	5·26	300999	26
35	650792	4·21	951476	1·05	699316	5·26	300684	25
36	651044	4·20	951412	1·05	699632	5·26	300368	24
37	651297	4·20	951349	1·06	699947	5·26	300053	23
38	651549	4·20	951286	1·06	700263	5·25	299737	22
39	651800	4·19	951222	1·06	700578	5·25	299422	21
40	652052	4·19	951159	1·06	700893	5·25	299107	20
41	9·652304	4·19	9·951096	1·06	9·701208	5·24	10·298792	19
42	652555	4·18	951032	1·06	701523	5·24	298477	18
43	652806	4·18	950968	1·06	701837	5·24	298163	17
44	653057	4·18	950905	1·06	702152	5·24	297848	16
45	653308	4·18	950841	1·06	702466	5·24	297534	15
46	653558	4·17	950778	1·06	702780	5·23	297220	14
47	653808	4·17	950714	1·06	703095	5·23	296905	13
48	654059	4·17	950650	1·06	703409	5·23	296591	12
49	654309	4·16	950586	1·06	703723	5·23	296277	11
50	654558	4·16	950522	1·07	704036	5·22	295964	10
51	9·654808	4·16	9·950458	1·07	9·704350	5·22	10·295650	9
52	655058	4·16	950394	1·07	704663	5·22	295337	8
53	655307	4·15	950330	1·07	704977	5·22	295023	7
54	655556	4·15	950266	1·07	705290	5·22	294710	6
55	655805	4·15	950202	1·07	705603	5·21	294397	5
56	656054	4·14	950138	1·07	705916	5·21	294084	4
57	656302	4·14	950074	1·07	706228	5·21	293772	3
58	656551	4·14	950010	1·07	706541	5·21	293459	2
59	656799	4·13	949945	1·07	706854	5·21	293146	1
60	657047	4·13	949881	1·07	707166	5·20	292834	0
	Cosine	D.	Sine	63°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·657047	4·13	9·949881	1·07	9·707166	5·20	10·292834	60
1	657295	4·13	949816	1·07	707478	5·20	292522	59
2	657542	4·12	949752	1·07	707790	5·20	292210	58
3	657790	4·12	949688	1·08	708102	5·20	291898	57
4	658037	4·12	949623	1·08	708414	5·19	291586	56
5	658284	4·12	949558	1·08	708726	5·19	291274	55
6	658531	4·11	949494	1·08	709037	5·19	290963	54
7	658778	4·11	949429	1·08	709349	5·19	290651	53
8	659025	4·11	949364	1·08	709660	5·19	290340	52
9	659271	4·10	949300	1·08	709971	5·18	290029	51
10	659517	4·10	949235	1·08	710282	5·18	289718	50
11	9·659763	4·10	9·949170	1·08	9·710593	5·18	10·289407	49
12	660009	4·09	949105	1·08	710904	5·18	289096	48
13	660255	4·09	949040	1·08	711215	5·18	288785	47
14	660501	4·09	948975	1·08	711525	5·17	288475	46
15	660746	4·09	948910	1·08	711836	5·17	288164	45
16	660991	4·08	948845	1·08	712146	5·17	287854	44
17	661236	4·08	948780	1·09	712456	5·17	287544	43
18	661481	4·08	948715	1·09	712766	5·16	287234	42
19	661726	4·07	948650	1·09	713070	5·16	286924	41
20	661970	4·07	948584	1·09	713386	5·16	286614	40
21	9·662214	4·07	9·948519	1·09	9·713696	5·16	10·286304	39
22	662459	4·07	948454	1·09	714005	5·16	285995	38
23	662703	4·06	948388	1·09	714314	5·15	285686	37
24	662946	4·06	948323	1·09	714624	5·15	285376	36
25	663190	4·06	948257	1·09	714933	5·15	285067	35
26	663433	4·05	948192	1·09	715242	5·15	284758	34
27	663677	4·05	948126	1·09	715551	5·14	284449	33
28	663920	4·05	948060	1·09	715860	5·14	284140	32
29	664163	4·05	947995	1·10	716168	5·14	283832	31
30	664406	4·04	947929	1·10	716477	5·14	283523	30
31	9·664648	4·04	9·947863	1·10	9·716785	5·14	10·283215	29
32	664891	4·04	947797	1·10	717093	5·13	282907	28
33	665133	4·03	947731	1·10	717401	5·13	282599	27
34	665375	4·03	947665	1·10	717709	5·13	282291	26
35	665617	4·03	947600	1·10	718017	5·13	281983	25
36	665859	4·02	947533	1·10	718325	5·13	281670	24
37	666100	4·02	947467	1·10	718633	5·12	281367	23
38	666342	4·02	947401	1·10	718940	5·12	281060	22
39	666583	4·02	947335	1·10	719248	5·12	280752	21
40	666824	4·01	947269	1·10	719555	5·12	280445	20
41	9·667065	4·01	9·947203	1·10	9·719862	5·12	10·280138	19
42	667305	4·01	947136	1·11	720169	5·11	279831	18
43	667546	4·01	947070	1·11	720476	5·11	279524	17
44	667786	4·00	947004	1·11	720783	5·11	279217	16
45	668027	4·00	946937	1·11	721089	5·11	278911	15
46	668267	4·00	946871	1·11	721396	5·11	278604	14
47	668506	3·99	946804	1·11	721702	5·10	278298	13
48	668746	3·99	946738	1·11	722009	5·10	277991	12
49	668986	3·99	946671	1·11	722315	5·10	277685	11
50	669225	3·99	946604	1·11	722621	5·10	277379	10
51	9·669464	3·98	9 946538	1·11	9·722927	5·10	10·277073	9
52	669703	3·98	946471	1·11	723232	5·09	276768	8
53	669942	3·98	946404	1·11	723538	5·09	276462	7
54	670181	3·97	946337	1·11	723844	5·09	276156	6
55	670419	3·97	946270	1·12	724149	5·09	275851	5
56	670658	3·97	946203	1·12	724454	5·09	275546	4
57	670896	3·97	946136	1·12	724759	5·08	275241	3
58	671134	3·96	946069	1·12	725065	5·08	274935	2
59	671372	3·96	946002	1·12	725369	5·08	274631	1
60	671609	3·96	945935	1·12	725674	5·08	274326	0
	Cosine	D.	Sine	62°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·671609	3·96	9·945935	1·12	9·725674	5·08	10·274326	60
1	671847	3·95	945868	1·12	725979	5·08	274021	59
2	672084	3·95	945800	1·12	726284	5·07	273716	58
3	672321	3·95	945733	1·12	726588	5·07	273412	57
4	672558	3·95	945666	1·12	726892	5·07	273108	56
5	672795	3·94	945598	1·12	727197	5·07	272803	55
6	673032	3·94	945531	1·12	727501	5·07	272499	54
7	673268	3·94	945464	1·13	727805	5·06	272195	53
8	673505	3·94	945396	1·13	728109	5·06	271891	52
9	673741	3·93	945328	1·13	728412	5·06	271588	51
10	673977	3·93	945261	1·13	728716	5·06	271284	50
11	9·674213	3·93	9·945193	1·13	9·729020	5·06	10·270980	49
12	674448	3·92	945125	1·13	729323	5·05	270677	48
13	674684	3·92	945058	1·13	729626	5·05	270374	47
14	674919	3·92	944990	1·13	729929	5·05	270071	46
15	675155	3·92	944922	1·13	730233	5·05	269767	45
16	675390	3·91	944854	1·13	730535	5·05	269465	44
17	675624	3·91	944786	1·13	730838	5·04	269162	43
18	675859	3·91	944718	1·13	731141	5·04	268859	42
19	676094	3·91	944650	1·13	731444	5·04	268556	41
20	676328	3·90	944582	1·14	731746	5·04	268254	40
21	9·676562	3·90	9·944514	1·14	9·732048	5·04	10·267952	39
22	676796	3·90	944446	1·14	732351	5·03	267649	38
23	677030	3·90	944377	1·14	732653	5·03	267347	37
24	677264	3·89	944309	1·14	732955	5·03	267045	36
25	677498	3·89	944241	1·14	733257	5·03	266743	35
26	677731	3·89	944172	1·14	733558	5·03	266442	34
27	677964	3·88	944104	1·14	733860	5·02	266140	33
28	678197	3·88	944036	1·14	734162	5·02	265838	32
29	678430	3·88	943967	1·14	734463	5·02	265537	31
30	678663	3·88	943899	1·14	734764	5·02	265236	30
31	9·678895	3·87	9·943830	1·14	9·735066	5·02	10·264934	29
32	679128	3·87	943761	1·14	735367	5·02	264633	28
33	679360	3·87	943693	1·15	735668	5·01	264332	27
34	679592	3·87	943624	1·15	735969	5·01	264031	26
35	679824	3·86	943555	1·15	736269	5·01	263731	25
36	680056	3·86	943486	1·15	736570	5·01	263430	24
37	680288	3·86	943417	1·15	736871	5·01	263129	23
38	680519	3·85	943348	1·15	737171	5·00	262829	22
39	680750	3·85	943279	1·15	737471	5·00	262529	21
40	680982	3·85	943210	1·15	737771	5·00	262229	20
41	9·681213	3·85	9·943141	1·15	9·738071	5·00	10·261929	19
42	681443	3·84	943072	1·15	738371	5·00	261629	18
43	681674	3·84	943003	1·15	738671	4·99	261329	17
44	681905	3·84	942934	1·15	738971	4·99	261029	16
45	682135	3·84	942864	1·15	739271	4·99	260729	15
46	682365	3·83	942795	1·16	739570	4·99	260430	14
47	682595	3·83	942726	1·16	739870	4·99	260130	13
48	682825	3·83	942656	1·16	740169	4·99	259831	12
49	683055	3·83	942587	1·16	740468	4·98	259532	11
50	683284	3·82	942517	1·16	740767	4·98	259233	10
51	9·683514	3·82	9·942448	1·16	9·741066	4·98	10·258934	9
52	683743	3·82	942378	1·16	741365	4·98	258635	8
53	683972	3·82	942308	1·16	741664	4·98	258336	7
54	684201	3·81	942239	1·16	741962	4·97	258038	6
55	684430	3·81	942169	1·16	742261	4·97	257739	5
56	684658	3·81	942099	1·16	742559	4·97	257441	4
57	684887	3·80	942029	1·16	742858	4·97	257142	3
58	685115	3·80	941959	1·16	743156	4·97	256844	2
59	685343	3·80	941889	1·17	743454	4·97	256546	1
60	685571	3·80	941819	1·17	743752	4·96	256248	0
	Cosine	D.	Sine	61°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·685571	3·80	9·941819	1·17	9·743752	4·96	10·256248	60
1	685799	3·79	941749	1·17	744050	4·96	255950	59
2	686027	3·79	941679	1·17	744348	4·96	255652	58
3	686254	3·79	941609	1·17	744645	4·96	255355	57
4	686482	3·79	941539	1·17	744943	4·96	255057	56
5	686709	3·78	941469	1·17	745240	4·96	254760	55
6	686936	3·78	941398	1·17	745538	4·95	254462	54
7	687163	3·78	941328	1·17	745835	4·95	254165	53
8	687389	3·78	941258	1·17	746132	4·95	253868	52
9	687616	3·77	941187	1·17	746429	4·95	253571	51
10	687843	3·77	941117	1·17	746726	4·95	253274	50
11	9·688069	3·77	9·941046	1·18	9·747023	4·94	10·252977	49
12	688295	3·77	940975	1·18	747319	4·94	252681	48
13	688521	3·76	940905	1·18	747616	4·94	252384	47
14	688747	3·76	940834	1·18	747913	4·94	252087	46
15	688972	3·76	940763	1·18	748209	4·94	251791	45
16	689198	3·76	940693	1·18	748505	4·93	251495	44
17	689423	3·75	940622	1·18	748801	4·93	251199	43
18	689648	3·75	940551	1·18	749097	4·93	250903	42
19	689873	3·75	940480	1·18	749393	4·93	250607	41
20	690098	3·75	940409	1·18	749689	4·93	250311	40
21	9·690323	3·74	9·940338	1·18	9·749985	4·93	10·250015	39
22	690548	3·74	940267	1·18	750281	4·92	249719	38
23	690772	3·74	940196	1·18	750576	4·92	249424	37
24	690996	3·74	940125	1·19	750872	4·92	249128	36
25	691220	3·73	940054	1·19	751167	4·92	248833	35
26	691444	3·73	939982	1·19	751462	4·92	248538	34
27	691668	3·73	939911	1·19	751757	4·92	248243	33
28	691892	3·73	939840	1·19	752052	4·91	247948	32
29	692115	3·72	939768	1·19	752347	4·91	247653	31
30	692339	3·72	939697	1·19	752642	4·91	247358	30
31	9·692562	3·72	9·939625	1·19	9·752937	4·91	10·247063	29
32	692785	3·71	939554	1·19	753231	4·91	246769	28
33	693008	3·71	939482	1·19	753526	4·91	246474	27
34	693231	3·71	939410	1·19	753820	4·90	246180	26
35	693453	3·71	939339	1·19	754115	4·90	245885	25
36	693676	3·70	939267	1·20	754409	4·90	245591	24
37	693898	3·70	939195	1·20	754703	4·90	245297	23
38	694120	3·70	939123	1·20	754997	4·90	245003	22
39	694342	3·70	939052	1·20	755291	4·90	244709	21
40	694564	3·69	938980	1·20	755585	4·89	244415	20
41	9·694786	3·69	9·938908	1·20	9·755878	4·89	10·244122	19
42	695007	3·69	938836	1·20	756172	4·89	243828	18
43	695229	3·69	938763	1·20	756465	4·89	243535	17
44	695450	3·68	938691	1·20	756759	4·89	243241	16
45	695671	3·68	938619	1·20	757052	4·89	242948	15
46	695892	3·68	938547	1·20	757345	4·88	242655	14
47	696113	3·68	938475	1·20	757638	4·88	242362	13
48	696334	3·67	938402	1·21	757931	4·88	242069	12
49	696554	3·67	938330	1·21	758224	4·88	241776	11
50	696775	3·67	938258	1·21	758517	4·88	241483	10
51	9·696995	3·67	9·938185	1·21	9·758810	4·88	10·241190	9
52	697215	3·66	938113	1·21	759102	4·87	240898	8
53	697435	3·66	938040	1·21	759395	4·87	240605	7
54	697654	3·66	937967	1·21	759687	4·87	240313	6
55	697874	3·66	937895	1·21	759979	4·87	240021	5
56	698094	3·65	937822	1·21	760272	4·87	239728	4
57	698313	3·65	937749	1·21	760564	4·87	239436	3
58	698532	3·65	937676	1·21	760856	4·86	239144	2
59	698751	3·65	937604	1·21	761148	4·86	238852	1
60	698970	3·64	937531	1·21	761439	4·86	238561	0
	Cosine	D.	Sine	60°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·698970	3·64	9·937531	1·21	9·761439	4·86	10·238561	60
1	699189	3·64	937458	1·22	761731	4·86	238269	59
2	699407	3·64	937385	1·22	762023	4·86	237977	58
3	699626	3·64	937312	1·22	762314	4·86	237686	57
4	699844	3·64	937238	1·22	762606	4·85	237394	56
5	700062	3·63	937165	1·22	762897	4·85	237103	55
6	700280	3·63	937092	1·22	763188	4·85	236812	54
7	700498	3·63	937019	1·22	763479	4·85	236521	53
8	700716	3·63	936946	1·22	763770	4·85	236230	52
9	700933	3·62	936872	1·22	764061	4·85	235939	51
10	701151	3·62	936799	1·22	764352	4·84	235648	50
11	9·701368	3·62	9·936725	1·22	9·764643	4·84	10·235357	49
12	701585	3·62	936652	1·23	764933	4·84	235067	48
13	701802	3·61	936578	1·23	765224	4·84	234776	47
14	702019	3·61	936505	1·23	765514	4·84	234486	46
15	702236	3·61	936431	1·23	765805	4·84	234195	45
16	702452	3·61	936357	1·23	766095	4·84	233905	44
17	702669	3·60	936284	1·23	766385	4·83	233615	43
18	702885	3·60	936210	1·23	766675	4·83	233325	42
19	703101	3·60	936136	1·23	766965	4·83	233035	41
20	703317	3·60	936062	1·23	767255	4·83	232745	40
21	9·703533	3·59	9·935988	1·23	9·767545	4·83	10·232455	39
22	703749	3·59	935914	1·23	767834	4·83	232166	38
23	703964	3·59	935840	1·23	768124	4·82	231876	37
24	704179	3·59	935766	1·24	768413	4·82	231587	36
25	704395	3·59	935692	1·24	768703	4·82	231297	35
26	704610	3·58	935618	1·24	768992	4·82	231008	34
27	704825	3·58	935543	1·24	769281	4·82	230719	33
28	705040	3·58	935469	1·24	769570	4·82	230430	32
29	705254	3·58	935395	1·24	769860	4·81	230140	31
30	705469	3·57	935320	1·24	770148	4·81	229852	30
31	9·705683	3·57	9·935246	1·24	9·770437	4·81	10·229563	29
32	705898	3·57	935171	1·24	770726	4·81	229274	28
33	706112	3·57	935097	1·24	771015	4·81	228985	27
34	706326	3·56	935022	1·24	771303	4·81	228697	26
35	706539	3·56	934948	1·24	771592	4·81	228408	25
36	706753	3·56	934873	1·24	771880	4·80	228120	24
37	706967	3·56	934798	1·25	772168	4·80	227832	23
38	707180	3·55	934723	1·25	772457	4·80	227543	22
39	707393	3·55	934649	1·25	772745	4·80	227255	21
40	707606	3·55	934574	1·25	773033	4·80	226967	20
41	9·707819	3·55	9·934499	1·25	9·773321	4·80	10·226679	19
42	708032	3·54	934424	1·25	773608	4·79	226392	18
43	708245	3·54	934349	1·25	773896	4·79	226104	17
44	708458	3·54	934274	1·25	774184	4·79	225816	16
45	708670	3·54	934199	1·25	774471	4·79	225529	15
46	708882	3·53	934123	1·25	774759	4·79	225241	14
47	709094	3·53	934048	1·25	775046	4·79	224954	13
48	709306	3·53	933973	1·25	775333	4·79	224667	12
49	709518	3·53	933898	1·26	775621	4·78	224379	11
50	709730	3·53	933822	1·26	775908	4·78	224092	10
51	9·709941	3·52	9·933747	1·26	9·776195	4·78	10·223805	9
52	710153	3·52	933671	1·26	776482	4·78	223518	8
53	710364	3·52	933596	1·26	776769	4·78	223231	7
54	710575	3·52	933520	1·26	777055	4·78	222945	6
55	710786	3·51	933445	1·26	777342	4·78	222658	5
56	710997	3·51	933369	1·26	777628	4·77	222372	4
57	711208	3·51	933293	1·26	777915	4·77	222085	3
58	711419	3·51	933217	1·26	778201	4·77	221799	2
59	711629	3·50	933141	1·26	778487	4·77	221512	1
60	711839	3·50	933066	1·26	778774	4·77	221226	0
	Cosine	D.	Sine	59°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·711839	3·50	9·933066	1·26	9·778774	4·77	10·221226	60
1	712050	3·50	932990	1·27	779060	4·77	220940	59
2	712260	3·50	932914	1·27	779346	4·76	220654	58
3	712469	3·49	932838	1·27	779632	4·76	220368	57
4	712679	3·49	932762	1·27	779918	4·76	220082	56
5	712889	3·49	932685	1·27	780203	4·76	219797	55
6	713098	3·49	932609	1·27	780489	4·76	219511	54
7	713308	3·49	932533	1·27	780775	4·76	219225	53
8	713517	3·48	932457	1·27	781060	4·76	218940	52
9	713726	3·48	932380	1·27	781346	4·75	218654	51
10	713935	3·48	932304	1·27	781631	4·75	218369	50
11	9·714144	3·48	9·932228	1·27	9·781916	4·75	10·218084	49
12	714352	3·47	932151	1·27	782201	4·75	217799	48
13	714561	3·47	932075	1·28	782486	4·75	217514	47
14	714769	3·47	931998	1·28	782771	4·75	217229	46
15	714978	3·47	931921	1·28	783056	4·75	216944	45
16	715186	3·47	931845	1·28	783341	4·75	216659	44
17	715394	3·46	931768	1·28	783626	4·74	216374	43
18	715602	3·46	931691	1·28	783910	4·74	216090	42
19	715809	3·46	931614	1·28	784195	4·74	215805	41
20	716017	3·46	931537	1·28	784479	4·74	215521	40
21	9·716224	3·45	9·931460	1·28	9·784764	4·74	10·215236	39
22	716432	3·45	931383	1·28	785048	4·74	214952	38
23	716639	3·45	931306	1·28	785332	4·73	214668	37
24	716846	3·45	931229	1·29	785616	4·73	214384	36
25	717053	3·45	931152	1·29	785900	4·73	214100	35
26	717259	3·44	931075	1·29	786184	4·73	213816	34
27	717466	3·44	930998	1·29	786468	4·73	213532	33
28	717673	3·44	930921	1·29	786752	4·73	213248	32
29	717879	3·44	930843	1·29	787036	4·73	212964	31
30	718085	3·43	930766	1·29	787319	4·72	212681	30
31	9·718291	3·43	9·930688	1·29	9·787603	4·72	10·212397	29
32	718497	3·43	930611	1·29	787886	4·72	212114	28
33	718703	3·43	930533	1·29	788170	4·72	211830	27
34	718909	3·43	930456	1·29	788453	4·72	211547	26
35	719114	3·42	930378	1·29	788736	4·72	211264	25
36	719320	3·42	930300	1·30	789019	4·72	210981	24
37	719525	3·42	930223	1·30	789302	4·71	210698	23
38	719730	3·42	930145	1·30	789585	4·71	210415	22
39	719935	3·41	930067	1·30	789868	4·71	210132	21
40	720140	3·41	929989	1·30	790151	4·71	209849	20
41	9·720345	3·41	9·929911	1·30	9·790433	4·71	10·209567	19
42	720549	3·41	929833	1·30	790716	4·71	209284	18
43	720754	3·40	929755	1·30	790999	4·71	209001	17
44	720958	3·40	929677	1·30	791281	4·71	208719	16
45	721162	3·40	929599	1·30	791563	4·70	208437	15
46	721366	3·40	929521	1·30	791846	4·70	208154	14
47	721570	3·40	929442	1·30	792128	4·70	207872	13
48	721774	3·39	929364	1·31	792410	4·70	207590	12
49	721978	3·39	929286	1·31	792692	4·70	207308	11
50	722181	3·39	929207	1·31	792974	4·70	207026	10
51	9·722385	3·39	9·929129	1·31	9·793256	4·70	10·206744	9
52	722588	3·39	929050	1·31	793538	4·69	206462	8
53	722791	3·38	928972	1·31	793819	4·69	206181	7
54	722994	3·38	928893	1·31	794101	4·69	205899	6
55	723197	3·38	928815	1·31	794383	4·69	205617	5
56	723400	3·38	928736	1·31	794664	4·69	205336	4
57	723603	3·37	928657	1·31	794945	4·69	205055	3
58	723805	3·37	928578	1·31	795227	4·69	204773	2
59	724007	3·37	928499	1·31	795508	4·68	204492	1
60	724210	3·37	928420	1·31	795789	4·68	204211	0
	Cosine	D.	Sine	**58°**	Cotang.	D.	Tang.	M.

(32 DEGREES.) A TABLE OF LOGARITHMIC

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·724210	3·37	9·928420	1·32	9·795789	4·68	10·204211	60
1	724412	3·37	928342	1·32	796070	4·68	203930	59
2	724614	3·36	928263	1·32	796351	4·68	203649	58
3	724816	3·36	928183	1·32	796632	4·68	203368	57
4	725017	3·36	928104	1·32	796913	4·68	203087	56
5	725219	3·36	928025	1·32	797194	4·68	202806	55
6	725420	3·35	927946	1·32	797475	4·68	202525	54
7	725622	3·35	927867	1·32	797755	4·68	202245	53
8	725823	3·35	927787	1·32	798036	4·67	201964	52
9	726024	3·35	927708	1·32	798316	4·67	201684	51
10	726225	3·35	927629	1·32	798596	4·67	201404	50
11	9·726426	3·34	9·927549	1·32	9·798877	4·67	10·201123	49
12	726626	3·34	927470	1·33	799157	4·67	200843	48
13	726827	3·34	927390	1·33	799437	4·67	200563	47
14	727027	3·34	927310	1·33	799717	4·67	200283	46
15	727228	3·34	927231	1·33	799997	4·66	200003	45
16	727428	3·33	927151	1·33	800277	4·66	199723	44
17	727628	3·33	927071	1·33	800557	4·66	199443	43
18	727828	3·33	926991	1·33	800836	4·66	199164	42
19	728027	3·33	926911	1·33	801116	4·66	198884	41
20	728227	3·33	926831	1·33	801396	4·66	198604	40
21	9·728427	3·32	9·926751	1·33	9·801675	4·66	10·198325	39
22	728626	3·32	926671	1·33	801955	4·66	198045	38
23	728825	3·32	926591	1·33	802234	4·65	197766	37
24	729024	3·32	926511	1·34	802513	4·65	197487	36
25	729223	3·31	926431	1·34	802792	4·65	197208	35
26	729422	3·31	926351	1·34	803072	4·65	196928	34
27	729621	3·31	926270	1·34	803351	4·65	196649	33
28	729820	3·31	926190	1·34	803630	4·65	196370	32
29	730018	3·30	926110	1·34	803908	4·65	196092	31
30	730216	3·30	926029	1·34	804187	4·65	195813	30
31	9·730415	3·30	9·925949	1·34	9·804466	4·64	10·195534	29
32	730613	3·30	925868	1·34	804745	4·64	195255	28
33	730811	3·30	925788	1·34	805023	4·64	194977	27
34	731009	3·29	925707	1·34	805302	4·64	194698	26
35	731206	3·29	925626	1·34	805580	4·64	194420	25
36	731404	3·29	925545	1·35	805859	4·64	194141	24
37	731602	3·29	925465	1·35	806137	4·64	193863	23
38	731799	3·29	925384	1·35	806415	4·63	193585	22
39	731996	3·28	925303	1·35	806693	4·63	193307	21
40	732193	3·28	925222	1·35	806971	4·63	193029	20
41	9·732390	3·28	9·925141	1·35	9·807249	4·63	10·192751	19
42	732587	3·28	925060	1·35	807527	4·63	192473	18
43	732784	3·28	924979	1·35	807805	4·63	192195	17
44	732980	3·27	924897	1·35	808083	4·63	191917	16
45	733177	3·27	924816	1·35	808361	4·63	191639	15
46	733373	3·27	924735	1·36	808638	4·62	191362	14
47	733569	3·27	924654	1·36	808916	4·62	191084	13
48	733765	3·27	924572	1·36	809193	4·62	190807	12
49	733961	3·26	924491	1·36	809471	4·62	190529	11
50	734157	3·26	924409	1·36	809748	4·62	190252	10
51	9·734353	3·26	9·924328	1·36	9·810025	4·62	10·189975	9
52	734549	3·26	924246	1·36	810302	4·62	189698	8
53	734744	3·25	924164	1·36	810580	4·62	189420	7
54	734939	3·25	924083	1·36	810857	4·62	189143	6
55	735135	3·25	924001	1·36	811134	4·61	188866	5
56	735330	3·25	923919	1·36	811410	4·61	188590	4
57	735525	3·25	923837	1·36	811687	4·61	188313	3
58	735719	3·24	923755	1·37	811964	4·61	188036	2
59	735914	3·24	923673	1·37	812241	4·61	187759	1
60	736109	3·24	923591	1·37	812517	4·61	187483	0
	Cosine	D.	Sine	57°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·736109	3·24	9·923591	1·37	9·812517	4·61	10·187482	60
1	736303	3·24	923509	1·37	812794	4·61	187206	59
2	736498	3·24	923427	1·37	813070	4·61	186930	58
3	736692	3·23	923345	1·37	813347	4·60	186653	57
4	736886	3·23	923263	1·37	813623	4·60	186377	56
5	737080	3·23	923181	1·37	813899	4·60	186101	55
6	737274	3·23	923098	1·37	814175	4·60	185825	54
7	737467	3·23	923016	1·37	814452	4·60	185548	53
8	737661	3·22	922933	1·37	814728	4·60	185272	52
9	737855	3·22	922851	1·37	815004	4·60	184996	51
10	738048	3·22	922768	1·38	815279	4·60	184721	50
11	9·738241	3·22	9·922686	1·38	9·815555	4·59	10·184445	49
12	738434	3·22	922603	1·38	815831	4·59	184169	48
13	738627	8·21	922520	1·38	816107	4·59	183893	47
14	738820	3·21	922438	1·38	816382	4·59	183618	46
15	739013	3·21	922355	1·38	816658	4·59	183342	45
16	739206	3·21	922272	1·38	816933	4·59	183067	44
17	739398	3·21	922189	1·38	817209	4·59	182791	43
18	739590	3·20	922106	1·38	817484	4·59	182516	42
19	739783	3·20	922023	1·38	817759	4·59	182241	41
20	739975	3·20	921940	1·38	818035	4·58	181965	40
21	9·740167	3·20	9·921857	1·39	9·818310	4·58	10·181690	39
22	740359	3·20	921774	1·39	818585	4·58	181415	38
23	740550	3·19	921691	1·39	818860	4·58	181140	37
24	740742	3·19	921607	1·39	819135	4·58	180865	36
25	740934	3·19	921524	1·39	819410	4·58	180590	35
26	741125	3·19	921441	1·39	819684	4·58	180316	34
27	741316	3·19	921357	1·39	819959	4·58	180041	33
28	741508	3·18	921274	1·39	820234	4·58	179766	32
29	741699	3·18	921190	1·39	820508	4·57	179492	31
30	741889	3·18	921107	1·39	820783	4·57	179217	30
31	9·742080	3·18	9·921023	1·39	9·821057	4·57	10·178943	29
32	742271	3·18	920939	1·40	821332	4·57	178668	28
33	742462	3·17	920856	1·40	821606	4·57	178394	27
34	742652	3·17	920772	1·40	821880	4·57	178120	26
35	742842	3·17	920688	1·40	822154	4·57	177846	25
36	743033	3·17	920604	1·40	822429	4·57	177571	24
37	743223	3·17	920520	1·40	822703	4·57	177297	23
38	743413	3·16	920436	1·40	822977	4·56	177023	22
39	743602	3·16	920352	1·40	823250	4·56	176750	21
40	743792	3·16	920268	1·40	823524	4·56	176476	20
41	9·743982	3·16	9·920184	1·40	9·823798	4·56	10·176202	19
42	744171	3·16	920099	1·40	824072	4·56	175928	18
43	744361	3·15	920015	1·40	824345	4·56	175655	17
44	744550	3·15	919931	1·41	824619	4·56	175381	16
45	744739	3·15	919846	1·41	824893	4·56	175107	15
46	744928	3·15	919762	1·41	825166	4·56	174834	14
47	745117	3·15	919677	1·41	825439	4·55	174561	13
48	745306	3·14	919593	1·41	825713	4·55	174287	12
49	745494	3·14	919508	1·41	825986	4·55	174014	11
50	745683	3·14	919424	1·41	826259	4·55	173741	10
51	9·745871	3·14	9·919339	1·41	9·826532	4·55	10·173468	9
52	746059	3·14	919254	1·41	826805	4·55	173195	8
53	746248	3·13	919169	1·41	827078	4·55	172922	7
54	746436	4·13	919085	1·41	827351	4·55	172649	6
55	746624	3·13	919000	1·41	827624	4·55	172376	5
56	746812	3·13	918915	1·42	827897	4·54	172103	4
57	746999	3·13	918830	1·42	828170	4·54	171830	3
58	747187	3·12	918745	1·42	828442	4·54	171558	2
59	747374	3·12	918659	1·42	828715	4·54	171285	1
60	747562	3·12	918574	1·42	828987	4·54	171013	0
	Cosine	D.	Sine	56°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·747562	3·12	9·918574	1·42	9·828987	4·54	10·171013	60
1	747749	3·12	918489	1·42	829260	4·54	170740	59
2	747936	3·12	918404	1·42	829532	4·54	170468	58
3	748123	3·11	918318	1·42	829805	4·54	170195	57
4	748310	3·11	918233	1·42	830077	4·54	169923	56
5	748497	3·11	918147	1·42	830349	4·53	169651	55
6	748683	3·11	918062	1·42	830621	4·53	169379	54
7	748870	3·11	917976	1·43	830893	4·53	169107	53
8	749056	3·10	917891	1·43	831165	4·53	168835	52
9	749243	3·10	917805	1·43	831437	4·53	168563	51
10	749429	3·10	917719	1·43	831709	4·53	168291	50
11	9·749615	3·10	9·917634	1·43	9·831981	4·53	10·168019	49
12	749801	3·10	917548	1·43	832253	4·53	167747	48
13	749987	3·09	917462	1·43	832525	4·53	167475	47
14	750172	3·09	917376	1·43	832796	4·53	167204	46
15	750358	3·09	917290	1·43	833068	4·52	166932	45
16	750543	3·09	917204	1·43	833339	4·52	166661	44
17	750729	3·09	917118	1·44	833611	4·52	166389	43
18	750914	3·08	917032	1·44	833882	4·52	166118	42
19	751099	3·08	916946	1·44	834154	4·52	165846	41
20	751284	3·08	916859	1·44	834425	4·52	165575	40
21	9·751469	3·08	9·916773	1·44	9·834696	4·52	10·165304	39
22	751654	3·08	916687	1·44	834967	4·52	165033	38
23	751839	3·08	916600	1·44	835238	4·52	164762	37
24	752023	3·07	916514	1·44	835509	4·52	164491	36
25	752208	3·07	916427	1·44	835780	4·51	164220	35
26	752392	3·07	916341	1·44	836051	4·51	163949	34
27	752576	3·07	916254	1·44	836322	4·51	163678	33
28	752760	3·07	916167	1·45	836593	4·51	163407	32
29	752944	3·06	916081	1·45	836864	4·51	163136	31
30	753128	3·06	915994	1·45	837134	4·51	162866	30
31	9·753312	3·06	9·915907	1·45	9·837405	4·51	10·162595	29
32	753495	3·06	915820	1·45	837675	4·51	162325	28
33	753679	3·06	915733	1·45	837946	4·51	162054	27
34	753862	3·05	915646	1·45	838216	4·51	161784	26
35	754046	3·05	915559	1·45	838487	4·50	161513	25
36	754229	3·05	915472	1·45	838757	4·50	161243	24
37	754412	3·05	915385	1·45	839027	4·50	160973	23
38	754595	3·05	915297	1·45	839297	4·50	160703	22
39	754778	3·04	915210	1·45	839568	4·50	160432	21
40	754960	3·04	915123	1·46	839838	4·50	160162	20
41	9·755143	3·04	9·915035	1·46	9·840108	4·50	10·159892	19
42	755326	3·04	914948	1·46	840378	4·50	159622	18
43	755508	3·04	914860	1·46	840647	4·50	159353	17
44	755690	3·04	914773	1·46	840917	4·49	159083	16
45	755872	3·03	914685	1·46	841187	4·49	158813	15
46	756054	3·03	914598	1·46	841457	4·49	158543	14
47	756236	3·03	914510	1·46	841726	4·49	158274	13
48	756418	3·03	914422	1·46	841996	4·49	158004	12
49	756600	3·03	914334	1·46	842266	4·49	157734	11
50	756782	3·02	914246	1·47	842535	4·49	157465	10
51	9·756963	3·02	9·914158	1·47	9·842805	4·49	10·157195	9
52	757144	3·02	914070	1·47	843074	4·49	156926	8
53	757326	3·02	913982	1·47	843343	4·49	156657	7
54	757507	3·02	913894	1·47	843612	4·49	156388	6
55	757688	3·01	913806	1·47	843882	4·48	156118	5
56	757869	3·01	913718	1·47	844151	4·48	155849	4
57	758050	3·01	913630	1·47	844420	4·48	155580	3
58	758230	3·01	913541	1·47	844689	4·48	155311	2
59	758411	3·01	913453	1·47	844958	4·48	155042	1
60	758591	3·01	913365	1·47	845227	4·48	154773	0
	Cosine	D.	Sine	55°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·758591	3·01	9·913365	1·47	9·845227	4·48	10·154773	60
1	758772	3·00	913276	1·47	845496	4·48	154504	59
2	758952	3·00	913187	1·48	845764	4·48	154236	58
3	759132	3·00	913099	1·48	846033	4·48	153967	57
4	759312	3·00	913010	1·48	846302	4·48	153698	56
5	759492	3·00	912922	1·48	846570	4·47	153430	55
6	759672	2·99	912833	1·48	846839	4·47	153161	54
7	759852	2·99	912744	1·48	847107	4·47	152893	53
8	760031	2·99	912655	1·48	847376	4·47	152624	52
9	760211	2·99	912566	1·48	847644	4·47	152356	51
10	760390	2·99	912477	1·48	847913	4·47	152087	50
11	9·760569	2·98	9·912388	1·48	9·848181	4·47	10·151819	49
12	760748	2·98	912299	1·49	848449	4·47	151551	48
13	760927	2·98	912210	1·49	848717	4·47	151283	47
14	761106	2·98	912121	1·49	848986	4·47	151014	46
15	761285	2·98	912031	1·49	849254	4·47	150746	45
16	761464	2·98	911942	1·49	849522	4·47	150478	44
17	761642	2·97	911853	1·49	849790	4·46	150210	43
18	761821	2·97	911763	1·49	850058	4·46	149942	42
19	761999	2·97	911674	1·49	850325	4·46	149675	41
20	762177	2·97	911584	1·49	850593	4·46	149407	40
21	9·762356	2·97	9·911495	1·49	9·850861	4·46	10·149139	39
22	762534	2·96	911405	1·49	851129	4·46	148871	38
23	762712	2·96	911315	1·50	851396	4·46	148604	37
24	762889	2·96	911226	1·50	851664	4·46	148336	36
25	763067	2·96	911136	1·50	851931	4·46	148069	35
26	763245	2·96	911046	1·50	852199	4·46	147801	34
27	763422	2·96	910956	1·50	852466	4·46	147534	33
28	763600	2·95	910866	1·50	852733	4·45	147267	32
29	763777	2·95	910776	1·50	853001	4·45	146999	31
30	763954	2·95	910686	1·50	853268	4·45	146732	30
31	9·764131	2·95	9·910596	1·50	9·853535	4·45	10·146465	29
32	764308	2·95	910506	1·50	853802	4·45	146198	28
33	764485	2·94	910415	1·50	854069	4·45	145931	27
34	764662	2·94	910325	1·51	854336	4·45	145664	26
35	764838	2·94	910235	1·51	854603	4·45	145397	25
36	765015	2·94	910144	1·51	854870	4·45	145130	24
37	765191	2·94	910054	1·51	855137	4·45	144863	23
38	765367	2·94	909963	1·51	855404	4·45	144596	22
39	765544	2·93	909873	1·51	855671	4·44	144329	21
40	765720	2·93	909782	1·51	855938	4·44	144062	20
41	9·765896	2·93	9·909691	1·51	9·856204	4·44	10·143796	19
42	766072	2·93	909601	1·51	856471	4·44	143529	18
43	766247	2·93	909510	1·51	856737	4·44	143263	17
44	766423	2·93	909419	1·51	857004	4·44	142996	16
45	766598	2·92	909328	1·52	857270	4·44	142730	15
46	766774	2·92	909237	1·52	857537	4·44	142463	14
47	766949	2·92	909146	1·52	857803	4·44	142197	13
48	767124	2·92	909055	1·52	858069	4·44	141931	12
49	767300	2·92	908964	1·52	858336	4·44	141664	11
50	767475	2·91	908873	1·52	858602	4·43	141398	10
51	9·767649	2·91	9·908781	1·52	9·858868	4·43	10·141132	9
52	767824	2·91	908690	1·52	859134	4·43	140866	8
53	767999	2·91	908599	1·52	859400	4·43	140600	7
54	768173	2·91	908507	1·52	859666	4·43	140334	6
55	768348	2·90	908416	1·53	859932	4·43	140068	5
56	768522	2·90	908324	1·53	860198	4·43	139802	4
57	768697	2·90	908233	1·53	860464	4·43	139536	3
58	768871	2·90	908141	1·53	860730	4·43	139270	2
59	769045	2·90	908049	1·53	860995	4·43	139005	1
60	769219	2·90	907958	1·53	861251	4·43	138739	0
	Cosine	D.	Sine	54°	Cotang	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·769219	2·90	9·907958	1·53	9·861261	4·43	10·138739	60
1	769393	2·89	907866	1·53	861527	4·43	138473	59
2	769566	2·89	907774	1·53	861792	4·42	138208	58
3	769740	2·89	907682	1·53	862058	4·42	137942	57
4	769913	2·89	907590	1·53	862323	4·42	137677	56
5	770087	2·89	907498	1·53	862589	4·42	137411	55
6	770260	2·88	907406	1·53	862854	4·42	137146	54
7	770433	2·88	907314	1·54	863119	4·42	136881	53
8	770606	2·88	907222	1·54	863385	4·42	136615	52
9	770779	2·88	907129	1·54	863650	4·42	136350	51
10	770952	2·88	907037	1·54	863915	4·42	136085	50
11	9·771125	2·88	9·906945	1·54	9·864180	4·42	10·135820	49
12	771298	2·87	906852	1·54	864445	4·42	135555	48
13	771470	2·87	906760	1·54	864710	4·42	135290	47
14	771643	2·87	906667	1·54	864975	4·41	135025	46
15	771815	2·87	906575	1·54	865240	4·41	134760	45
16	771987	2·87	906482	1·54	865505	4·41	134495	44
17	772159	2·87	906389	1·55	865770	4·41	134230	43
18	772331	2·86	906296	1·55	866035	4·41	133965	42
19	772503	2·86	906204	1·55	866300	4·41	133700	41
20	772675	2·86	906111	1·55	866564	4·41	133436	40
21	9·772847	2·86	9·906018	1·55	9·866829	4·41	10·133171	39
22	773018	2·86	905925	1·55	867094	4·41	132906	38
23	773190	2·86	905832	1·55	867358	4·41	132642	37
24	773361	2·85	905739	1·55	867623	4·41	132377	36
25	773533	2·85	905645	1·55	867887	4·41	132113	35
26	773704	2·85	905552	1·55	868152	4·40	131848	34
27	773875	2·85	905459	1·55	868416	4·40	131584	33
28	774046	2·85	905366	1·56	868680	4·40	131320	32
29	774217	2·85	905272	1·56	868945	4·40	131055	31
30	774388	2·84	905179	1·56	869209	4·40	130794	30
31	9·774558	2·84	9·905085	1·56	9·869473	4·40	10·130527	29
32	774729	2·84	904992	1·56	869737	4·40	130263	28
33	774899	2·84	904898	1·56	870001	4·40	129999	27
34	775070	2·84	904804	1·56	870265	4·40	129735	26
35	775240	2·84	904711	1·56	870529	4·40	129471	25
36	775410	2·83	904617	1·56	870793	4·40	129207	24
37	775580	2·83	904523	1·56	871057	4·40	128943	23
38	775750	2·83	904429	1·57	871321	4·40	128679	22
39	775920	2·83	904335	1·57	871585	4·40	128415	21
40	776090	2·83	904241	1·57	871849	4·39	128151	20
41	9·776259	2·83	9·904147	1·57	9·872112	4·39	10·127888	19
42	776429	2·82	904053	1·57	872376	4·39	127624	18
43	776598	2·82	903959	1·57	872640	4·39	127360	17
44	776768	2·82	903864	1·57	872903	4·39	127097	16
45	776937	2·82	903770	1·57	873167	4·39	126833	15
46	777106	2·82	903676	1·57	873430	4·39	126570	14
47	777275	2·81	903581	1·57	873694	4·39	126306	13
48	777444	2·81	903487	1·57	873957	4·39	126043	12
49	777613	2·81	903392	1·58	874220	4·39	125780	11
50	777781	2·81	903298	1·58	874484	4·39	125516	10
51	9·777950	2·81	9·903203	1·58	9·874747	4·39	10·125253	9
52	778119	2·81	903108	1·58	875010	4·39	124990	8
53	778287	2·80	903014	1·58	875273	4·38	124727	7
54	778455	2·80	902919	1·58	875536	4·38	124464	6
55	778624	2·80	902824	1·58	875800	4·38	124200	5
56	778792	2·80	902729	1·58	876063	4·38	123937	4
57	778960	2·80	902634	1·58	876326	4·38	123674	3
58	779128	2·80	902539	1·59	876589	4·38	123411	2
59	779295	2·79	902444	1·59	876851	4·38	123149	1
60	779463	2·79	902349	1·59	877114	4·38	122886	0
	Cosine	D.	Sine	53°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·779463	2·79	9·902349	1·59	9·877114	4·38	10·122886	60
1	779631	2·79	902253	1·59	877377	4·38	122623	59
2	779798	2·79	902158	1·59	877640	4·38	122360	58
3	779966	2·79	902063	1·59	877903	4·38	122097	57
4	780133	2·79	901967	1·59	878165	4·38	121835	56
5	780300	2·78	901872	1·59	878428	4·38	121572	55
6	780467	2·78	901776	1·59	878691	4·38	121309	54
7	780634	2·78	901681	1·59	878953	4·37	121047	53
8	780801	2·78	901585	1·59	879216	4·37	120784	52
9	780968	2·78	901490	1·59	879478	4·37	120522	51
10	781134	2·78	901394	1·60	879741	4·37	120259	50
11	9·781301	2·77	9·901298	1·60	9·880003	4·37	10·119997	49
12	781468	2·77	901202	1·60	880265	4·37	119735	48
13	781634	2·77	901106	1·60	880528	4·37	119472	47
14	781800	2·77	901010	1·60	880790	4·37	119210	46
15	781966	2·77	900914	1·60	881052	4·37	118948	45
16	782132	2·77	900818	1·60	881314	4·37	118686	44
17	782298	2·76	900722	1·60	881576	4·37	118424	43
18	782464	2·76	900626	1·60	881839	4·37	118161	42
19	782630	2·76	900529	1·60	882101	4·37	117899	41
20	782796	2·76	900433	1·61	882363	4·36	117637	40
21	9·782961	2·76	9·900337	1·61	9·882625	4·36	10·117375	39
22	783127	2·76	900240	1·61	882887	4·36	117113	38
23	783292	2·75	900144	1·61	883148	4·36	116852	37
24	783458	2·75	900047	1·61	883410	4·36	116590	36
25	783623	2·75	899951	1·61	883672	4·36	116328	35
26	783788	2·75	899854	1·61	883934	4·36	116066	34
27	783953	2·75	899757	1·61	884196	4·36	115804	33
28	784118	2·75	899660	1·61	884457	4·36	115543	32
29	784282	2·74	899564	1·61	884719	4·36	115281	31
30	784447	2·74	899467	1·62	884980	4·36	115020	30
31	9·784612	2·74	9·899370	1·62	9·885242	4·36	10·114758	29
32	784776	2·74	899273	1·62	885503	4·36	114497	28
33	784941	2·74	899176	1·62	885765	4·36	114235	27
34	785105	2·74	899078	1·62	886026	4·36	113974	26
35	785269	2·73	898981	1·62	886288	4·36	113712	25
36	785433	2·73	898884	1·62	886549	4·35	113451	24
37	785597	2·73	898787	1·62	886810	4·35	113190	23
38	785761	2·73	898689	1·62	887072	4·35	112928	22
39	785925	2·73	898592	1·62	887333	4·35	112667	21
40	786089	2·73	898494	1·63	887594	4·35	112406	20
41	9·786252	2·72	9·898397	1·63	9·887855	4·35	10·112145	19
42	786416	2·72	898299	1·63	888116	4·35	111884	18
43	786579	2·72	898202	1·63	888377	4·35	111623	17
44	786742	2·72	898104	1·63	888639	4·35	111361	16
45	786906	2·72	898006	1·63	888900	4·35	111100	15
46	787069	2·72	897908	1·63	889160	4·35	110840	14
47	787232	2·71	897810	1·63	889421	4·35	110579	13
48	787395	2·71	897712	1·63	889682	4·35	110318	12
49	787557	2·71	897614	1·63	889943	4·35	110057	11
50	787720	2·71	897516	1·63	890204	4·34	109796	10
51	9·787883	2·71	9·897418	1·64	9·890465	4·34	10·109535	9
52	788045	2·71	897320	1·64	890725	4·34	109275	8
53	788208	2·71	897222	1·64	890986	4·34	109014	7
54	788370	2·70	897123	1·64	891247	4·34	108753	6
55	788532	2·70	897025	1·64	891507	4·34	108493	5
56	788694	2·70	896926	1·64	891768	4·34	108232	4
57	788856	2·70	896828	1·64	892028	4·34	107972	3
58	789018	2·70	896729	1·64	892289	4·34	107711	2
59	789180	2·70	896631	1·64	892549	4·34	107451	1
60	789342	2·69	896532	1·64	892810	4·34	107190	0
	Cosine	D.	Sine	52°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·789342	2·69	9·896532	1·64	9·892810	4·34	10·107190	60
1	789504	2·69	896433	1·65	893070	4·34	106930	59
2	789665	2·69	896335	1·65	893331	4·34	106669	58
3	789827	2·69	896236	1·65	893591	4·34	106409	57
4	789988	2·69	896137	1·65	893851	4·34	106149	56
5	790149	2·69	896038	1·65	894111	4·34	105889	55
6	790310	2·68	895939	1·65	894371	4·34	105629	54
7	790471	2·68	895840	1·65	894632	4·33	105368	53
8	790632	2·68	895741	1·65	894892	4·33	105108	52
9	790793	2·68	895641	1·65	895152	4·33	104848	51
10	790954	2·68	895542	1·65	895412	4·33	104588	50
11	9·791115	2·68	9·895443	1·66	9·895672	4·33	10·104328	49
12	791275	2·67	895343	1·66	895932	4·33	104068	48
13	791436	2·67	895244	1·66	896192	4·33	103808	47
14	791596	2·67	895145	1·66	896452	4·33	103548	46
15	791757	2·67	895045	1·66	896712	4·33	103288	45
16	791917	2·67	894945	1·66	896971	4·33	103029	44
17	792077	2·67	894846	1·66	897231	4·33	102769	43
18	792237	2·66	894746	1·66	897491	4·33	102509	42
19	792397	2·66	894646	1·66	897751	4·33	102249	41
20	792557	2·66	894546	1·66	898010	4·33	101990	40
21	9·792716	2·66	9·894446	1·67	9·898270	4·33	10·101730	39
22	792876	2·66	894346	1·67	898530	4·33	101470	38
23	793035	2·66	894246	1·67	898789	4·33	101211	37
24	793195	2·65	894146	1·67	899049	4·32	100951	36
25	793354	2·65	894046	1·67	899308	4·32	100692	35
26	793514	2·65	893946	1·67	899568	4·32	100432	34
27	793673	2·65	893846	1·67	899827	4·32	100173	33
28	793832	2·65	893745	1·67	900086	4·32	099914	32
29	793991	2·65	893645	1·67	900346	4·32	099654	31
30	794150	2·64	893544	1·67	900605	4·32	099395	30
31	9·794308	2·64	9·893444	1·68	9·900864	4·32	10·099136	29
32	794467	2·64	893343	1·68	901124	4·32	098876	28
33	794626	2·64	893243	1·68	901383	4·32	098617	27
34	794784	2·64	893142	1·68	901642	4·32	098358	26
35	794942	2·64	893041	1·68	901901	4·32	098099	25
36	795101	2·64	892940	1·68	902160	4·32	097840	24
37	795259	2·63	892839	1·68	902419	4·32	097581	23
38	795417	2·63	892739	1·68	902679	4·32	097321	22
39	795575	2·63	892638	1·68	902938	4·32	097062	21
40	795733	2·63	892536	1·68	903197	4·31	096803	20
41	9·795891	2·63	9·892435	1·69	9·903455	4·31	10·096545	19
42	796049	2·63	892334	1·69	903714	4·31	096286	18
43	796206	2·63	892233	1·69	903973	4·31	096027	17
44	796364	2·62	892132	1·69	904232	4·31	095768	16
45	796521	2·62	892030	1·69	904491	4·31	095509	15
46	796679	2·62	891929	1·69	904750	4·31	095250	14
47	796836	2·62	891827	1·69	905008	4·31	094992	13
48	796993	2·62	891726	1·69	905267	4·31	094733	12
49	797150	2·61	891624	1·69	905526	4·31	094474	11
50	797307	2·61	891523	1·70	905784	4·31	094216	10
51	9·797464	2·61	9·891421	1·70	9·906043	4·31	10·093957	9
52	797621	2·61	891319	1·70	906302	4·31	093698	8
53	797777	2·61	891217	1·70	906560	4·31	093440	7
54	797934	2·61	891115	1·70	906819	4·31	093181	6
55	798091	2·61	891013	1·70	907077	4·31	092923	5
56	798247	2·61	890911	1·70	907336	4·31	092664	4
57	798403	2·60	890809	1·70	907594	4·31	092406	3
58	798560	2·60	890707	1·70	907852	4·31	092148	2
59	798716	2·60	890605	1·70	908111	4·30	091889	1
60	798872	2·60	890503	1·70	908369	4·30	091631	0
	Cosine	D.	Sine	51°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·798872	2·60	9·890503	1·70	9·908369	4·30	10·091631	60
1	799028	2·60	890400	1·71	908628	4·30	091372	59
2	799184	2·60	890298	1·71	908886	4·30	091114	58
3	799339	2·59	890195	1·71	909144	4·30	090856	57
4	799495	2·59	890093	1·71	909402	4·30	090598	56
5	799651	2·59	889990	1·71	909660	4·30	090340	55
6	799806	2·59	889888	1·71	909918	4·30	000082	54
7	799962	2·59	889785	1·71	910177	4·30	089823	53
8	800117	2·59	889682	1·71	910435	4·30	089565	52
9	800272	2·58	889579	1·71	910693	4·30	089307	51
10	800427	2·58	889477	1·71	910951	4·30	089049	50
11	9·800582	2·58	9·889374	1·72	9·911209	4·30	10·088791	49
12	800737	2·58	889271	1·72	911467	4·30	088533	48
13	800892	2·58	889168	1·72	911724	4·30	088276	47
14	801047	2·58	889064	1·72	911982	4·30	088018	46
15	801201	2·58	888961	1·72	912240	4·30	087760	45
16	801356	2·57	888858	1·72	912498	4·30	087502	44
17	801511	2·57	888755	1·72	912756	4·30	087244	43
18	801665	2·57	888651	1·72	913014	4·29	086986	42
19	801819	2·57	888548	1·72	913271	4·29	086729	41
20	801973	2·57	888444	1·73	913529	4·29	086471	40
21	9·802128	2·57	9·888341	1·73	9·913787	4·29	10·086213	39
22	802282	2·56	888237	1·73	914044	4·29	085956	38
23	802436	2·56	888134	1·73	914302	4·29	085698	37
24	802589	2·56	888030	1·73	914560	4·29	085440	36
25	802743	2·56	887926	1·73	914817	4·29	085183	35
26	802897	2·56	887822	1·73	915075	4·29	084925	34
27	803050	2·56	887718	1·73	915332	4·29	084668	33
28	803204	2·56	887614	1·73	915590	4·29	084410	32
29	803357	2·55	887510	1·73	915847	4·29	084153	31
30	803511	2·55	887406	1·74	916104	4·29	083896	30
31	9·803664	2·55	9·887302	1·74	9·916362	4·29	10·083638	29
32	803817	2·55	887198	1·74	916619	4·29	083381	28
33	803970	2·55	887093	1·74	916877	4·29	083123	27
34	804123	2·55	886989	1·74	917134	4·29	082866	26
35	804276	2·54	886885	1·74	917391	4·29	082609	25
36	804428	2·54	886780	1·74	917648	4·29	082352	24
37	804581	2·54	886676	1·74	917905	4·29	082095	23
38	804734	2·54	886571	1·74	918163	4·28	081837	22
39	804886	2·54	886466	1·74	918420	4·28	081580	21
40	805039	2·54	886362	1·75	918677	4·28	081323	20
41	9·805191	2·54	9·886257	1·75	9·918934	4·28	10·081066	19
42	805343	2·53	886152	1·75	919191	4·28	080809	18
43	805495	2·53	886047	1·75	919448	4·28	080552	17
44	805647	2·53	885942	1·75	919705	4·28	080295	16
45	805799	2·53	885837	1·75	919962	4·28	080038	15
46	805951	2·53	885732	1·75	920219	4·28	079781	14
47	806103	2·53	885627	1·75	920476	4·28	079524	13
48	806254	2·53	885522	1·75	920733	4·28	079267	12
49	806406	2·52	885416	1·75	920990	4·28	079010	11
50	806557	2·52	885311	1·76	921247	4·28	078753	10
51	9·806709	2·52	9·885205	1·76	9·921503	4·28	10·078497	9
52	806860	2·52	885100	1·76	921760	4·28	078240	8
53	807011	2·52	884994	1·76	922017	4·28	077983	7
54	807163	2·52	884889	1·76	922274	4·28	077726	6
55	807314	2·52	884783	1·76	922530	4·28	077470	5
56	807465	2·51	884677	1·76	922787	4·28	077213	4
57	807615	2·51	884572	1·76	923044	4·28	076956	3
58	807766	2·51	884466	1·76	923300	4·28	076700	2
59	807917	2·51	884360	1·76	923557	4·27	076443	1
60	808067	2·51	884254	1·77	923813	4·27	076187	0
	Cosine	D.	Sine	50°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·808067	2·51	9·884254	1·77	9·923813	4·27	10·076187	60
1	808218	2·51	884148	1·77	924070	4·27	075930	59
2	808368	2·51	884042	1·77	924327	4·27	075673	58
3	808519	2·50	883936	1·77	924583	4·27	075417	57
4	808669	2·50	883829	1·77	924840	4·27	075160	56
5	808819	2·50	883723	1·77	925096	4·27	074904	55
6	808969	2·50	883617	1·77	925352	4·27	074648	54
7	809119	2·50	883510	1·77	925609	4·27	074391	53
8	809269	2·50	883404	1·77	925865	4·27	074135	52
9	809419	2·49	883297	1·78	926122	4·27	073878	51
10	809569	2·49	883191	1·78	926378	4·27	073622	50
11	9·809718	2·49	9·883084	1·78	9·926634	4·27	10·073366	49
12	809868	2·49	882977	1·78	926890	4·27	073110	48
13	810017	2·49	882871	1·78	927147	4·27	072853	47
14	810167	2·49	882764	1·78	927403	4·27	072597	46
15	810316	2·48	882657	1·78	927659	4·27	072341	45
16	810465	2·48	882550	1·78	927915	4·27	072085	44
17	810614	2·48	882443	1·78	928171	4·27	071829	43
18	810763	2·48	882336	1·79	928427	4·27	071573	42
19	810912	2·48	882229	1·79	928683	4·27	071317	41
20	811061	2·48	882121	1·79	928940	4·27	071060	40
21	9·811210	2·48	9·882014	1·79	9·929196	4·27	10·070804	39
22	811358	2·47	881907	1·79	929452	4·27	070548	38
23	811507	2·47	881799	1·79	929708	4·27	070292	37
24	811655	2·47	881692	1·79	929964	4·26	070036	36
25	811804	2·47	881584	1·79	930220	4·26	069780	35
26	811952	2·47	881477	1·79	930475	4·26	069525	34
27	812100	2·47	881369	1·79	930731	4·26	069269	33
28	812248	2·47	881261	1·80	930987	4·26	069013	32
29	812396	2·46	881153	1·80	931243	4·26	068757	31
30	812544	2·46	881046	1·80	931499	4·26	068501	30
31	9·812692	2·46	9·880938	1·80	9·931755	4·26	10·068245	29
32	812840	2·46	880830	1·80	932010	4·26	067990	28
33	812988	2·46	880722	1·80	932266	4·26	067734	27
34	813135	2·46	880613	1·80	932522	4·26	067478	26
35	813283	2·46	880505	1·80	932778	4·26	067222	25
36	813430	2·45	880397	1·80	933033	4·26	066967	24
37	813578	2·45	880289	1·81	933289	4·26	066711	23
38	813725	2·45	880180	1·81	933545	4·26	066455	22
39	813872	2·45	880072	1·81	933800	4·26	066200	21
40	814019	2·45	879963	1·81	934056	4·26	065944	20
41	9·814166	2·45	9·879855	1·81	9·934311	4·26	10·065689	19
42	814313	2·45	879746	1·81	934567	4·26	065433	18
43	814460	2·44	879637	1·81	934823	4·26	065177	17
44	814607	2·44	879529	1·81	935078	4·26	064922	16
45	814753	2·44	879420	1·81	935333	4·26	064667	15
46	814900	2·44	879311	1·81	935589	4·26	064411	14
47	815046	2·44	879202	1·82	935844	4·26	064156	13
48	815193	2·44	879093	1·82	936100	4·26	063900	12
49	815339	2·44	878984	1·82	936355	4·26	063645	11
50	815485	2·43	878875	1·82	936610	4·26	063390	10
51	9·815631	2·43	9·878766	1·82	9·936866	4·25	10·063134	9
52	815778	2·43	878656	1·82	937121	4·25	062879	8
53	815924	2·43	878547	1·82	937376	4·25	062624	7
54	816069	2·43	878438	1·82	937632	4·25	062368	6
55	816215	2·43	878328	1·82	937887	4·25	062113	5
56	816361	2·43	878219	1·83	938142	4·25	061858	4
57	816507	2·42	878109	1·83	938398	4·25	061602	3
58	816652	2·42	877999	1·83	938653	4·25	061347	2
59	816798	2·42	877890	1·83	938908	4·25	061092	1
60	816943	2·42	877780	1·83	939163	4·25	060837	0
	Cosine	D.	Sine	49°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·816943	2·42	9·877780	1·83	9·939163	4·25	10·060837	60
1	817088	2·42	877670	1·83	939418	4·25	060582	59
2	817233	2·42	877560	1·83	939673	4·25	060327	58
3	817379	2·42	877450	1·83	939928	4·25	060072	57
4	817524	2·41	877340	1·83	940183	4·25	059817	56
5	817668	2·41	877230	1·84	940438	4·25	059562	55
6	817813	2·41	877120	1·84	940694	4·25	059306	54
7	817958	2·41	877010	1·84	940949	4·25	059051	53
8	818103	2·41	876899	1·84	941204	4·25	058796	52
9	818247	2·41	876789	1·84	941458	4·25	058542	51
10	818392	2·41	876678	1·84	941714	4·25	058286	50
11	9·818536	2·40	9·876568	1·84	9·941968	4·25	10·058032	49
12	818681	2·40	876457	1·84	942223	4·25	057777	48
13	818825	2·40	876347	1·84	942478	4·25	057522	47
14	818969	2·40	876236	1·85	942733	4·25	057267	46
15	819113	2·40	876125	1·85	942988	4·25	057012	45
16	819257	2·40	876014	1·85	943243	4·25	056757	44
17	819401	2·40	875904	1·85	943498	4·25	056502	43
18	819545	2·39	875793	1·85	943752	4·25	056248	42
19	819689	2·39	875682	1·85	944007	4·25	055993	41
20	819832	2·39	875571	1·85	944262	4·25	055738	40
21	9·819976	2·39	9·875459	1·85	9·944517	4·25	10·055483	39
22	820120	2·39	875348	1·85	944771	4·24	055229	38
23	820263	2·39	875237	1·85	945026	4·24	054974	37
24	820406	2·39	875126	1·86	945281	4·24	054719	36
25	820550	2·38	875014	1·86	945535	4·24	054465	35
26	820693	2·38	874903	1·86	945790	4·24	054210	34
27	820836	2·38	874791	1·86	946045	4·24	053955	33
28	820979	2·38	874680	1·86	946299	4·24	053701	32
29	821122	2·38	874568	1·86	946554	4·24	053446	31
30	821265	2·38	874456	1·86	946808	4·24	053192	30
31	9·821407	2·38	9·874344	1·86	9·947063	4·24	10·052937	29
32	821550	2·38	874232	1·87	947318	4·24	052682	28
33	821693	2·37	874121	1·87	947572	4·24	052428	27
34	821835	2·37	874009	1·87	947826	4·24	052174	26
35	821977	2·37	873896	1·87	948081	4·24	051919	25
36	822120	2·37	873784	1·87	948336	4·24	051664	24
37	822262	2·37	873672	1·87	948590	4·24	051410	23
38	822404	2·37	873560	1·87	948844	4·24	051156	22
39	822546	2·37	873448	1·87	949099	4·24	050901	21
40	822688	2·36	873335	1·87	949353	4·24	050647	20
41	9·822830	2·36	9·873223	1·87	9·949607	4·24	10·050393	19
42	822972	2·36	873110	1·88	949862	4·24	050138	18
43	823114	2·36	872998	1·88	950116	4·24	049884	17
44	823255	2·36	872885	1·88	950370	4·24	049630	16
45	823397	2·36	872772	1·88	950625	4·24	049375	15
46	823539	2·36	872659	1·88	950879	4·24	049121	14
47	823680	2·35	872547	1·88	951133	4·24	048867	13
48	823821	2·35	872434	1·88	951388	4·24	048612	12
49	823963	2·35	872321	1·88	951642	4·24	048358	11
50	824104	2·35	872208	1·88	951896	4·24	048104	10
51	9·824245	2·35	9·872095	1·89	9·952150	4·24	10·047850	9
52	824386	2·35	871981	1·89	952405	4·24	047595	8
53	824527	2·35	871868	1·89	952659	4·24	047341	7
54	824668	2·34	871755	1·89	952913	4·24	047087	6
55	824808	2·34	871641	1·89	953167	4·23	046833	5
56	824949	2·34	871528	1·89	953421	4·23	046579	4
57	825090	2·34	871414	1·89	953675	4·23	046325	3
58	825230	2·34	871301	1·89	953929	4·23	046071	2
59	825371	2·34	871187	1·89	954183	4·23	045817	1
60	825511	2·34	871073	1·90	954437	4·23	045563	0
	Cosine	D.	Sine	48°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·825511	2·34	9·871073	1·90	9·954437	4·23	10·045563	60
1	825651	2·33	870960	1·90	954691	4·23	045309	59
2	825791	2·33	870846	1·90	954945	4·23	045055	58
3	825931	2·33	870732	1·90	955200	4·23	044800	57
4	826071	2·33	870618	1·90	955454	4·23	044546	56
5	826211	2·33	870504	1·90	955707	4·23	044293	55
6	826351	2·33	870390	1·90	955961	4·23	044039	54
7	826491	2·33	870276	1·90	956215	4·23	043785	53
8	826631	2·33	870161	1·90	956469	4·23	043531	52
9	826770	2·32	870047	1·91	956723	4·23	043277	51
10	826910	2·32	869933	1·91	956977	4·23	043023	50
11	9·827049	2·32	9·869818	1·91	9·957231	4·23	10·042769	49
12	827189	2·32	869704	1·91	957485	4·23	042515	48
13	827328	2·32	869589	1·91	957739	4·23	042261	47
14	827467	2·32	869474	1·91	957993	4·23	042007	46
15	827606	2·32	869360	1·91	958246	4·23	041754	45
16	827745	2·32	869245	1·91	958500	4·23	041500	44
17	827884	2·31	869130	1·91	958754	4·23	041246	43
18	828023	2·31	869015	1·92	959008	4·23	040992	42
19	828162	2·31	868900	1·92	959262	4·23	040738	41
20	828301	2·31	868785	1·92	959516	4·23	040484	40
21	9·828439	2·31	9·868670	1·92	9·959769	4·23	10·040231	39
22	828578	2·31	868555	1·92	960023	4·23	039977	38
23	828716	2·31	868440	1·92	960277	4·23	039723	37
24	828855	2·30	868324	1·92	960531	4·23	039469	36
25	828993	2·30	868209	1·92	960784	4·23	039216	35
26	829131	2·30	868093	1·92	961038	4·23	038962	34
27	829269	2·30	867978	1·93	961291	4·23	038709	33
28	829407	2·30	867862	1·93	961545	4·23	038455	32
29	829545	2·30	867747	1·93	961799	4·23	038201	31
30	829683	2·30	867631	1·93	962052	4·23	037948	30
31	9·829821	2·29	9·867515	1·93	9·962306	4·23	10·037694	29
32	829959	2·29	867399	1·93	962560	4·23	037440	28
33	830097	2·29	867283	1·93	962813	4·23	037187	27
34	830234	2·29	867167	1·93	963067	4·23	036933	26
35	830372	2·29	867051	1·93	963320	4·23	036680	25
36	830509	2·29	866935	1·94	963574	4·23	036426	24
37	830646	2·29	866819	1·94	963827	4·23	036173	23
38	830784	2·29	866703	1·94	964081	4·23	035919	22
39	830921	2·28	866586	1·94	964335	4·23	035665	21
40	831058	2·28	866470	1·94	964588	4·22	035412	20
41	9·831195	2·28	9·866353	1·94	9·964842	4·22	10·035158	19
42	831332	2·28	866237	1·94	965095	4·22	034905	18
43	831469	2·28	866120	1·94	965349	4·22	034651	17
44	831606	2·28	866004	1·95	965602	4·22	034398	16
45	831742	2·28	865887	1·95	965855	4·22	034145	15
46	831879	2·28	865770	1·95	966105	4·22	033891	14
47	832015	2·27	865653	1·95	966362	4·22	033638	13
48	832152	2·27	865536	1·95	966616	4·22	033384	12
49	832288	2·27	865419	1·95	966869	4·22	033131	11
50	832425	2·27	865302	1·95	967123	4·22	032877	10
51	9·832561	2·27	9·865185	1·95	9·967376	4·22	10·032624	9
52	832697	2·27	865068	1·95	967629	4·22	032371	8
53	832833	2·27	864950	1·95	967883	4·22	032117	7
54	832969	2·26	864833	1·96	968136	4·22	031864	6
55	833105	2·26	864716	1·96	968389	4·22	031611	5
56	833241	2·26	864598	1·96	968643	4·22	031357	4
57	833377	2·26	864481	1·96	968896	4·22	031104	3
58	833512	2·26	864363	1·96	969149	4·22	030851	2
59	833648	2·26	864245	1·96	969403	4·22	030597	1
60	833783	2·26	864127	1·96	969656	4·22	030344	0
	Cosine	D.	Sine	47°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·833783	2·26	9·864127	1·96	9·969656	4·22	10·030344	60
1	833919	2·25	864010	1·96	969909	4·22	030091	59
2	834054	2·25	863892	1·97	970162	4·22	029838	58
3	834189	2·25	863774	1·97	970416	4·22	029584	57
4	834325	2·25	863656	1·97	970669	4·22	029331	56
5	834460	2·25	863538	1·97	970922	4·22	029078	55
6	834595	2·25	863419	1·97	971175	4·22	028825	54
7	834730	2·25	863301	1·97	971429	4·22	028571	53
8	834865	2·25	863183	1·97	971682	4·22	028318	52
9	834999	2·24	863064	1·97	971935	4·22	028065	51
10	835134	2·24	862946	1·98	972188	4·22	027812	50
11	9·835269	2·24	9·862827	1·98	9·972441	4·22	10·027559	49
12	835403	2·24	862709	1·98	972694	4·22	027306	48
13	835538	2·24	862590	1·98	972948	4·22	027052	47
14	835672	2·24	862471	1·98	973201	4·22	026799	46
15	835807	2·24	862353	1·98	973454	4·22	026546	45
16	835941	2·24	862234	1·98	973707	4·22	026293	44
17	836075	2·23	862115	1·98	973960	4·22	026040	43
18	836209	2·23	861996	1·98	974213	4·22	025787	42
19	836343	2·23	861877	1·98	974466	4·22	025534	41
20	836477	2·23	861758	1·99	974719	4·22	025281	40
21	9·836611	2·23	9·861638	1·99	9·974973	4·22	10·025027	39
22	836745	2·23	861519	1·99	975226	4·22	024774	38
23	836878	2·23	861400	1·99	975479	4·22	024521	37
24	837012	2·22	861280	1·99	975732	4·22	024268	36
25	837146	2·22	861161	1·99	975985	4·22	024015	35
26	837279	2·22	861041	1·99	976238	4·22	023762	34
27	837412	2·22	860922	1·99	976491	4·22	023509	33
28	837546	2·22	860802	1·99	976744	4·22	023256	32
29	837679	2·22	860682	2·00	976997	4·22	023003	31
30	837812	2·22	860562	2·00	977250	4·22	022750	30
31	9·837945	2·22	9·860442	2·00	9·977503	4·22	10·022497	29
32	838078	2·21	860322	2·00	977756	4·22	022244	28
33	838211	2·21	860202	2·00	978009	4·22	021991	27
34	838344	2·21	860082	2·00	978262	4·22	021738	26
35	838477	2·21	859962	2·00	978515	4·22	021485	25
36	838610	2·21	859842	2·00	978768	4·22	021232	24
37	838742	2·21	859721	2·01	979021	4·22	020979	23
38	838875	2·21	859601	2·01	979274	4·22	020726	22
39	839007	2·21	859480	2·01	979527	4·22	020473	21
40	839140	2·20	859360	2·01	979780	4·22	020220	20
41	9·839272	2·20	9·859239	2·01	9·980033	4·22	10·019967	19
42	839404	2·20	859119	2·01	980286	4·22	019714	18
43	839536	2·20	858998	2·01	980538	4·22	019462	17
44	839668	2·20	858877	2·01	980791	4·21	019209	16
45	839800	2·20	858756	2·02	981044	4·21	018956	15
46	839932	2·20	858635	2·02	981297	4·21	018703	14
47	840064	2·19	858514	2·02	981550	4·21	018450	13
48	840196	2·19	858393	2·02	981803	4·21	018197	12
49	840328	2·19	858272	2·02	982056	4·21	017944	11
50	840459	2·19	858151	2·02	982309	4·21	017691	10
51	9·840591	2·19	9·858029	2·02	9·982562	4·21	10·017438	9
52	840722	2·19	857908	2·02	982814	4·21	017186	8
53	840854	2·19	857786	2·02	983067	4·21	016933	7
54	840985	2·19	857665	2·03	983320	4·21	016680	6
55	841116	2·18	857543	2·03	983573	4·21	016427	5
56	841247	2·18	857422	2·03	983826	4·21	016174	4
57	841378	2·18	857300	2·03	984079	4·21	015921	3
58	841509	2·18	857178	2·03	984331	4·21	015669	2
59	841640	2·18	857056	2·03	984584	4·21	015416	1
60	841771	2·18	856934	2·03	984837	4·21	015163	0
	Cosine	D.	Sine	46°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·841771	2·18	9·856934	2·03	9·984837	4·21	10·015163	60
1	841902	2·18	856812	2·03	985090	4·21	014910	59
2	842033	2·18	856690	2·04	985343	4·21	014657	58
3	842163	2·17	856568	2·04	985596	4·21	014404	57
4	842294	2·17	856446	2·04	985848	4·21	014152	56
5	842424	2·17	856323	2·04	986101	4·21	013899	55
6	842555	2·17	856201	2·04	986354	4·21	013646	54
7	842685	2·17	856078	2·04	986607	4·21	013393	53
8	842815	2·17	855956	2·04	986860	4·21	013140	52
9	842946	2·17	855833	2·04	987112	4·21	012888	51
10	843076	2·17	855711	2·05	987365	4·21	012635	50
11	9·843206	2·16	9·855588	2·05	9·987618	4·21	10·012382	49
12	843336	2·16	855465	2·05	987871	4·21	012129	48
13	843466	2·16	855342	2·05	988123	4·21	011877	47
14	843595	2·16	855219	2·05	988376	4·21	011624	46
15	843725	2·16	855096	2·05	988629	4·21	011371	45
16	843855	2·16	854973	2·05	988882	4·21	011118	44
17	843984	2·16	854850	2·05	989134	4·21	010866	43
18	844114	2·15	854727	2·06	989387	4·21	010613	42
19	844243	2·15	854603	2·06	989640	4·21	010360	41
20	844372	2·15	854480	2·06	989893	4·21	010107	40
21	9·844502	2·15	9·854356	2·06	9·990145	4·21	10·009855	39
22	844631	2·15	854233	2·06	990398	4·21	009602	38
23	844760	2·15	854109	2·06	990651	4·21	009349	37
24	844889	2·15	853986	2·06	990903	4·21	009097	36
25	845018	2·15	853862	2·06	991156	4·21	008844	35
26	845147	2·15	853738	2·06	991409	4·21	008591	34
27	845276	2·14	853614	2·07	991662	4·21	008338	33
28	845405	2·14	853490	2·07	991914	4·21	008086	32
29	845533	2·14	853366	2·07	992167	4·21	007833	31
30	845662	2·14	853242	2·07	992420	4·21	007580	30
31	9·845790	2·14	9·853118	2·07	9·992672	4·21	10·007328	29
32	845919	2·14	852994	2·07	992925	4·21	007075	28
33	846047	2·14	852869	2·07	993178	4·21	006822	27
34	846175	2·14	852745	2·07	993430	4·21	006570	26
35	846304	2·14	852620	2·07	993683	4·21	006317	25
36	846432	2·13	852496	2·08	993936	4·21	006064	24
37	846560	2·13	852371	2·08	994189	4·21	005811	23
38	846688	2·13	852247	2·08	994441	4·21	005559	22
39	846816	2·13	852122	2·08	994694	4·21	005306	21
40	846944	2·13	851997	2·08	994947	4·21	005053	20
41	9·847071	2·13	9·851872	2·08	9·995199	4·21	10·004801	19
42	847199	2·13	851747	2·08	995452	4·21	004548	18
43	847327	2·13	851622	2·08	995705	4·21	004295	17
44	847454	2·12	851497	2·09	995957	4·21	004043	16
45	847582	2·12	851372	2·09	996210	4·21	003790	15
46	847709	2·12	851246	2·09	996463	4·21	003537	14
47	847836	2·12	851121	2·09	996715	4·21	003285	13
48	847964	2·12	850996	2·09	996968	4·21	003032	12
49	848091	2·12	850870	2·09	997221	4·21	002779	11
50	848218	2·12	850745	2·09	997473	4·21	002527	10
51	9·848345	2·12	9·850619	2·09	9·997726	4·21	10·002274	9
52	848472	2·11	850493	2·10	997979	4·21	002021	8
53	848599	2·11	850368	2·10	998231	4·21	001769	7
54	848726	2·11	850242	2·10	998484	4·21	001516	6
55	848852	2·11	850116	2·10	998737	4·21	001263	5
56	848979	2·11	849990	2·10	998989	4·21	001011	4
57	849106	2·11	849864	2·10	999242	4·21	000758	3
58	849232	2·11	849738	2·10	999495	4·21	000505	2
59	849359	2·11	849611	2·10	999748	4·21	000253	1
60	849485	2·11	849485	2·10	10·000000	4·21	10·000000	0
	Cosine	D.	Sine	45°	Cotang.	D.	Tang.	M.

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